This is a PDF of a manuscript that has been peer-reviewed and accepted for publication. As the article has not yet been formatted, copy edited or proofread, the final published version may be different from the early online release.

This pre-publication manuscript may be downloaded, distributed and used under the provisions of the Creative Commons Attribution 4.0 International (CC BY 4.0) license. It may be cited using the DOI below.

The DOI for this manuscript is
DOI:10.2151/jmsj.2021-050

J-STAGE Advance published date: April 30th, 2021
The final manuscript after publication will replace the preliminary version at the above DOI once it is available.
A 1D Bayesian inversion applied to GPM Microwave Imager observations: sensitivity studies

Marylis Barreyat
CNRM, Université de Toulouse, Météo-France, CNRS

Philippe Chambon
CNRM, Université de Toulouse, Météo-France, CNRS

Jean-François Mahfouf
CNRM, Université de Toulouse, Météo-France, CNRS

Ghislain Faure
CNRM, Université de Toulouse, Météo-France, CNRS

Yasutaka Ikuta
Meteorological Research Institute, Department of Observation & Data Assimilation; Japan Meteorological Agency, Numerical Prediction Division
Corresponding author: Philippe Chambon, CNRM, Météo-France and CNRS, Université de Toulouse, France.
E-mail: philippe.chambon@meteo.fr
Abstract

The assimilation of cloudy and rainy microwave observations is under investigation at Météo-France with a method called ‘1D-Bay+3D/4D-Var’. This method consists of two steps: (i) a Bayesian inversion of microwave observations and (ii) the assimilation of the retrieved relative humidity profiles in a 3D/4D-Var framework. In this paper, two estimators for the Bayesian inversion are used: either a weighted average (WA) or the maximum likelihood (ML) of a kernel density function. Sensitivity studies over the first step of the method are conducted for different degrees of freedom: the observation error, the channel selection and the scattering properties of frozen hydrometeors in the observation operator. Observations over a two-month period of the Global Precipitation Measurement (GPM) Microwave Imager (GMI) on-board the GPM-Core satellite and forecasts of the convective scale model Application of Research to Operations at Mesoscale (AROME) have been chosen to conduct these studies. Two different meteorological situations are analysed: those predicted cloudy in AROME but clear in the observations and, on the contrary, those predicted clear in AROME but cloudy in the observations. Main conclusions are as follows. First, low observational errors tend to be associated with the profiles with the highest consistency with the observations. Second, the validity of the retrieved profiles varies vertically with the set of channels used. Third, the radiative properties used
in the radiative transfer simulations have a strong influence on the retrieved atmospheric profiles. Finally, the ML estimator has the advantage of being independent of the observation error but is less constrained than the WA estimator when few frequencies are considered. While the presented sensitivities have been conducted to incorporate the scheme in a data assimilation system, the results may be generalized for geophysical retrieval purposes.
1. Introduction

In the mid-1990s, satellite observations began to be assimilated into Numerical Weather Prediction (NWP) models using radiative transfer codes. However, only clear-sky observations could be processed due to the non-linearity of the required observation operators, the non-normality and large variances of representativeness retrievals and other issues listed in Errico et al. (2007). A next step has been to assimilate information from cloudy and rainy microwave radiances. Early studies focused on the assimilation of satellite-derived rainfall rates and showed significant improvements in weather forecasting (Treadon et al. 2002; Marécal and Mahfouf 2002; Hou et al. 2004; Mahfouf et al. 2005 and Hou and Zhang 2007). These methods required different algorithms to convert satellite radiances in rain amounts for each instrument and were therefore abandoned in favor of a gradual transition towards the use of raw brightness temperatures. Then, in 2005, the European Centre for Medium-Range Weather Forecasts (ECMWF) started to assimilate both cloudy and rainy microwave radiances of the Special Sensor Microwave / Imager (SSM/I) instrument thanks to a 1D+4D Var method (Bauer et al. 2006a, 2006b; Geer et al. 2010, 2017). Such approach required the development of fast radiative transfer modeling with
scattering processes, together with their linearized versions, for variational data assimilation. In 2009 this methodology was replaced by a direct assimilation method accounting for model and representativeness errors and also led to better results in their 4D-Var system (Geer and Bauer 2010, 2011).

Bayesian inversions have a long history in being used for extracting information on atmospheric hydrometeors from microwave observations (e.g. Kummerow et al., 1996). Building on this heritage, a method named ’1D-Bay+3D-Var’ to assimilate cloudy and rainy microwave observations, was first used at Météo-France with ground based weather radar reflectivities (Caumont el al. 2010; Wattrelot et al. 2014). This method seeks to retrieve an optimal relative humidity profile, from an observation and short-term forecast profiles in its vinicity, thanks to a Bayesian inversion. Then retrieved profiles of relative humidity are assimilated into a 3D-Var system. This variable was preferred over the assimilation of the specific humidity and temperature profiles in order to directly impact the saturation of the atmosphere and modulate cloudiness in the forecast, as well as to facilitate the specification of observation errors. Among the advantages of this method with respect to direct assimilation of cloudy and rainy radiances, which is now more commonly used in the NWP context (Geer et al., 2017), one can mention that: (i) it does not rely on tangent linear and adjoint versions
of the observation operator, (ii) it does not suffer from the zero-gradient problem and can generate a cloud where no cloud is present in the first guess by saturating the atmosphere. A potential drawback of the method is that the retrieval to be assimilated can be correlated with the first guess but to a smaller extent than with a 1D-Var+4D-Var (Geer et al., 2008) thanks to a mitigation strategy further explained in Section 3. The Japan Meteorological Agency (JMA) successfully implemented a similar method, with a 4D-Var system, to operationally assimilate observations from space-borne radars (Ikuta and Honda, 2011; Ikuta, 2016). More recently, studies have been undertaken at Météo-France with the use of the 1D-Bay+4D-Var method with satellite radiances from the Sondeur Atmosphérique du Profil d’Humidité Intertropicale par Radiométrie (SAPHIR) instrument on-board the Megha-Tropiques satellite (Roca et al. 2020). This water vapor sounder has 6 frequencies all centered around the 183 GHz and is currently assimilated in the Action de Recherche Petite Echelle Grande Echelle (ARPEGE) global operational NWP model at Météo-France (Courtier et al. 1991) only in clear-sky areas (Chambon et al. 2015). Preliminary results show that cloudy and rainy SAPHIR observations improve wind, temperature and relative humidity forecasts in the ARPEGE model (Duruisseau et al. 2019). These improvements result from the assimilation of retrieved relative humidities at four different pressure levels. These encouraging studies lead
to a desire of extending the method to all microwave instruments. Since the 183 GHz frequency only probes solid hydrometeors, the use of a wider range of microwave channels will allow to observe liquid hydrometeors as well. The present paper therefore focuses on generalizing the results of Druisseau et al. (2019) to a larger set of frequencies, as well as investigating various sensitivities of the Bayesian inversion.

In February 2014, the Global Precipitation Measurement Core Observatory (GPM-Core) satellite mission was launched to provide a new standard for nearly global measurements of liquid and solid precipitation from the Tropics to the Mid-latitudes (Hou et al. 2014). To that end, it was equipped with two instruments: the Dual-frequency Precipitation Radar (DPR) with two frequencies (13.6 and 35.55 GHz) and the GMI radiometer. The latter is a microwave radiometer with a wide range of frequencies sounding water vapor as well as solid and liquid hydrometeors. ECMWF started to directly assimilate GMI clear and cloudy radiances into the 4D-Var system in 2017 (Lean et al. 2017; Geer et al. 2017). The Global Modeling and Assimilation Office (GMAO) of the National Aeronautics and Space Administration (NASA) started the all-sky assimilation as well in July 2018 (Kim et al. 2020). Météo-France assimilates GMI radiances as well, but in clear sky conditions only.

Due to its frequency diversity, the GMI instrument is well suited to study
the extension of the '1D-Bay + 3D/4D-Var' method to all microwave instruments and complement the preliminary studies done with SAPHIR. The present paper describes sensitivity studies regarding the first step of this method, the Bayesian inversion, with the GMI instrument. This inversion involves several parameters for which the impact of their prescribed values must be evaluated in order to improve the method.

The paper is organized as follows: section 2 describes the NWP data and the GMI observations selected to perform the sensitivity studies, section 3 presents the Bayesian inversion method and the metrics used to evaluate its retrieved results. In section 4 are shown sensitivity results of the inversion on three degrees of freedom: (i) the specification of the observation error, (ii) the channel selection and (iii) the specification of radiative transfer scattering properties within the observation operator. Finally, section 5 provides a number of conclusions from the sensitivity studies with recommendations on future activities.

2. GMI Data and NWP System

2.1 Satellite radiances

The level 1B products of the GMI instrument (GPM GMI L1B ATBD, 2016) have been chosen for conducting the sensitivity studies to be de-
scribed hereafter. This radiometer is on-board the Low Earth Orbiting
satellite GPM-Core Observatory which is orbiting around the Earth at an
altitude of 407 km on an inclined orbit of 65° with respect to the Equator.
The GMI instrument is characterized by a comprehensive set of channels,
summarized in Table 1, from 10.65 to 183.31±7 GHz. The dataset of inter-
est spans a two-month period from September 1st to October 31th, 2017.
From this dataset, only a geographical domain over the North Atlantic
Ocean is selected, corresponding to the domain of the NWP limited area
model described in the next section.

The channels have different footprint sizes that have been remapped on
a common grid to simplify the inversion. Hence, the raw Level 1B data
have been superobbed onto a regular lat/lon grid at 0.1° deg resolution,
compatible with the effective resolution of the NWP model used in this
study.

2.2 Numerical Weather Prediction (NWP) model

At Météo-France, the convective-scale NWP model AROME (Application
of Research to Operations at Mesoscale) is used operationally over numer-
ous geographical domains with forecasts ranging up to 48 hours (Seity et al.
2011). This model has a non-hydrostatic spectral dynamical core, explicitly
resolving deep moist convection; and is characterized by a finite difference
representation on the vertical with a discretization on 90 levels from the surface to 10 hPa. In the midlatitudes, Météo-France operates AROME over Western Europe at 1.3 km horizontal resolution; this version is initialised with a 3D-Var data assimilation system (Brousseau et al. 2016) and forced on its lateral boundaries with forecasts from the Météo-France global model ARPEGE. In the Tropics, Météo-France operates AROME at 2.5 km horizontal resolution over five geographical domains covering several French Overseas territories. This tropical version, noted afterwards AROME-OM, does not have a dedicated data assimilation system in operations yet. Instead, the AROME-OM versions are initialised with ECMWF analyses every 6 hours. The full description of the AROME-OM system can be found in Faure et al. (2020).

In this paper, the AROME-Antilles version, one of the AROME-OM models has been chosen, it covers the geographical domain [10.4° N to 22.45° N; 67.8° W to 52.2° W]. Even though the horizontal resolution of computation is on a 2.5 km grid, numerical models typically have an effective resolution three to four times larger than the original grid (Ricard et al. 2013). This leads to a typical effective resolution of 10 km which roughly corresponds to the chosen 0.1° superobbing resolution for GMI observations. The selected GMI superobbed observations are collocated with a 3-hour range AROME forecast, using a ± 90 min time tolerance, with the objective to build up
a 3-hour 3D-Var for AROME in a future study. Note that the results presented below are robust of a change of the first guess forecast range (not shown).

2.3 Observation operator

The observation operator used in the NWP model AROME in order to simulate brightness temperatures is the Radiative Transfer for TIROS Operational Vertical sounder code (RTTOV, Saunders et al. 2018). Within RTTOV, a module named RTTOV-SCATT, taking into account the scattering effect by hydrometeors based on the Delta-Eddington approximation, allows to simulate observations from microwave sensors within clouds (Bauer et al. 2006; Geer and Baordo, 2014). This study considers the version 12 of RTTOV-SCATT to simulate GMI’s Brightness Temperatures. Radiative properties necessary to simulate scattering effects from hydrometeors are listed in Table 2.

Table 2

Regarding the radiative properties of snowfall, Geer and Baordo (2014) showed that within the context of the ECMWF model, the best compromise across the different microwave frequencies affected by scattering was to choose the particle size distribution proposed by Field et al. (2007) together with the single scattering properties of a sector snowflake. This configura-
tion is selected to be the baseline for the present study. The other particle shapes from the scattering database of Liu (2008) are also considered in the sensivity studies described below.

3. Method

3.1 Bayesian Inversion

The first step of the ‘1D-Bay+4D-Var’, based on the Bayes’ theorem, performs a one dimensional retrieval of the atmospheric state, based on observations and on prior information from a short-term forecast also named First-Guess (FG). The retrieved state includes hydrometeors as well as humidity and temperature profiles.

Considering a set of observations \( y_o \) (GMI radiances), one wants to compute the probability to estimate the true state of the atmosphere: \( P(x = x_{true}|y = y_o) \). The \( x_{true} \) vector is the ‘real’ atmospheric profile. Thanks to the Bayes theorem, this probability can be rewritten as:

\[
P(x = x_{true}|y = y_o) \propto P(y = y_o|x = x_{true})P(x = x_{true})
\]

In practice, it is assumed that observation errors and FG (prior) errors are
not correlated and follow a Gaussian distribution i.e.:

\[ P(y = y_o|x = x_{true}) \propto e^{-\frac{1}{2}(y_o - H(x))^t R^{-1}(y_o - H(x))} \]

Where \( H \) is the observation operator, which simulates brightness temperatures from an atmospheric profile, i.e. \( H(x) = y \) and \( R \) is the covariance matrix of observation errors. Several assumptions are made to simplify the \( R \) matrix: (i) a unique value is considered for the variances and noted \( \sigma^2 \) in the following, (ii) the off-diagonal terms are set to zero, neglecting the interchannel correlations. This experimental \( R \) matrix is a first approach to be further improved in the future to take into account better the specifications of each frequency.

To estimate \( x_{true} \), one needs to define a database of atmospheric profiles for which \( P(x = x_{true}|y = y_o) \) values can be computed. The database used in the Bayesian inversion is made of FG profiles within the vicinity of a given observation. To limit error correlations between the retrieved profile and the first guess, which could be problematic for assimilation (Errico et al., 2007; Geer at al., 2008), the model profile at observation location is removed from the inversion database. It is assumed that these profiles have high \( P(x = x_{true}) \) values because they are consistent in terms of weather regime with the meteorological situation of interest. In addition, the oc-
currence and intensity of the cloud and precipitation profiles are assumed to be similar in the FG database and in nature. Therefore, the probability \( P(x = x_{true}) \) can be represented by the relative number of occurrences of profiles \( x_i \) in the database, i.e. by \( 1/n \), where \( n \) is the size of the selected database. Several mathematical estimators of \( x_{true} \) can be considered. In this study, we focus on two of them: either by selecting the \( x \) atmospheric state which is associated with the highest value of \( P(x = x_{true}|y = y_o) \). Such an estimator is used at JMA (Ikuta and Honda, 2011) and was used as well at Météo-France with airborne cloud radar observations and provided good results (Borderies et al. 2018). This estimator is known as the Maximum Likelihood estimator and noted hereafter ML.

Another estimator is the expected value (mean) using the full database as a source of information (Caumont et al. 2010), leading to the following expression: \( x_{ret} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \), where \( x_{ret} \) is the retrieved atmospheric profile from which we then extract the relative humidity profile. In that case, the mean estimator is a weighted average of the FG profiles available in the database:

\[
x_{ret} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}
\]
With
\[ w_i = \frac{1}{d} \sum_{j=1}^{d} \left( -\frac{1}{2} [y_{o,j} - H_j(x_i) - b_j]^T R^{-1} [y_{o,j} - H_j(x_i) - b_j] \right), \]
where \( i \) corresponds to a given profile in the inversion database (therefore not including the first guess as explained above), \( j \) a given channel within the selected ones, \( d \) the number of channels selected for the inversion and \( b_j \) a clear sky bias correction between the FG and the observations previously computed for each GMI frequencies over a two-month period (not shown). The values can be found Table 1. This estimator is noted hereafter WA for weighted average.

3.2 Definition of sensitivity studies

The mathematical expressions of the probabilities in the Bayesian inversion include several tunable parameters: (i) the specification of observation errors, (ii) the channel selection, (iii) the specification of radiative properties within the observation operator and (iv) the definition of the database. In addition, for each parameter, the choice of the estimator (WA or ML) may lead to a different result as well.

The database for each inversion consists of 625 atmospheric columns within a 250 × 250 km² square surrounding each observation (25 × 25 profiles); the
atmospheric columns are taken from 3-h forecasts of the AROME-Antilles model. Enlarging the database or changing the forecast range do not significantly modify the retrieval results (not shown); therefore, the sensitivity experiments hereafter focus only on the first three parameters.

Table 3 summarizes the range of values for the three different degrees of freedom of interest. The experiments have been designed to change a single parameter at a time from a reference configuration and examine the impact of each of them. The set up of the reference configuration has been chosen to be close to the GMI Noise Equivalent Delta Temperature ($Nc\Delta T$) and as well as to be consistent with previous studies for the observation error specification of the Bayesian inversion (Guerbette et al. 2016 and Duruisseau et al. 2019). A single value of observation error was selected for all channels for simplicity, and limit the numbers of sensitivity experiments. However, it is likely that optimal observation errors would vary from one channel to another and could be derived with dedicated diagnostics (e.g. Desroziers et al., 2005). Desroziers et al. 2005 first diagnostics performed (not shown) indicate that observation errors explored for the sensitivity study range between 1K and 20K (see table 3). The set up is also based on the willingness to use the full capability of the GMI sensor for the channel selection, and based on the literature for the specification of the radiative transfer model.
Note that the 10 GHz channels have been excluded from the study because the 0.1° superobbing resolution is not consistent with its coarser resolution (20 × 30 km). Furthermore, radiative transfer modelling of this particular frequency is known to suffer from deficiencies (Lean et al. 2017).

### 3.3 Metrics used for evaluating the inversions

The Bayesian inversion described before is designed to assimilate microwave radiances in clouds and precipitating systems. To identify these regions, a scattering index based on the normalized difference of brightness temperatures (Bt) between the two polarisations of the 37 GHz frequency is chosen (Petty and Katsaros, 1990; Petty, 1994, Geer and Bauer, 2010). Its expression is given by:

\[
P_{37} = 1 - \frac{Bt_{37v}^{\text{Clear}} - Bt_{37h}^{\text{Clear}}}{Bt_{37v}^{\text{Clear}} - Bt_{37h}^{\text{Clear}}}
\]

With \( Bt_{\text{Clear}} \) corresponding to the brightness temperatures simulated without taking into account the hydrometeors. This index can be computed both for the observations and the FG. In the case of observations, the denominator is taken as the same as for the FG (Geer and Bauer, 2010). A clear scene is assumed when \( P_{37} \) is below of 0.3 (Geer and Bauer, 2010).
and a cloudy scene is assumed when it exceeds this threshold. In this study, two situations are considered: a situation named $FG_{clear}O_{cloud}$ where the profiles taken into account are clear in the FG but cloudy in the observations and the opposite situation named $FG_{cloud}O_{clear}$ containing the cloudy profiles in the FG but clear in the observations. These two situations are critical in all-sky assimilation because they have the potential to correct for large prediction errors. They will also allow to examine how the Bayesian inversion can moisten or dry the FG when the retrievals are assimilated. For the two-month period, each category contains 3218 and 8440 observations respectively for the $FG_{cloud}O_{clear}$ and $FG_{clear}O_{cloud}$ (see Table 4). The number of meteorological scenes not treated in this paper is given in Table 4.

The results are compared in two different spaces: the model space and the observation space. Hence two different metrics are defined:

- The first one compares the brightness temperatures simulated with the retrieved profiles against GMI observations. The differences of brightness temperatures are examined in terms of correlation coefficient, bias and standard deviation of their distributions. Note that when the same channels are used within the inversion and in the comparisons, the derived statistics should not be seen as an independent validation. This evaluation can be considered as a sanity check similar to the examination of analysis de-
partures in a variational context that should be reduced with respect to background departures.

- The second one compares the retrieved relative humidity profiles to the FG relative humidity profiles. The differences of relative humidities are examined in terms of mean and standard deviation of their distributions.

4. Results from sensitivity studies

4.1 First-Guess Statistics

In order to correctly interpret the results of the Bayesian inversion, the diagnostics between the simulated FG brightness temperatures and the GMI observations are calculated over the two-month period for the $FG_{\text{cloud}O_{\text{clear}}}$ scenes (Figures 1-i) and the $FG_{\text{clear}O_{\text{cloud}}}$ scenes (Figures 1-ii). Figure 1-a shows the correlation coefficients between the simulated FG brightness temperatures and the GMI observations. These correlations range from 0 to 0.4 for the $FG_{\text{cloud}O_{\text{clear}}}$ scenes. The 23.8V GHz and $183.31 \pm 7$ GHz channels have the highest values whereas the 36.64V GHz and 89H GHz channels, on the contrary, have the lowest coefficients. For $FG_{\text{clear}O_{\text{cloud}}}$ scenes, the coefficients never exceed 0.6. The 23.8V GHz channel has the highest value and the 89H GHz channel has the lowest one. This may be explained by the sensitivity of the 23 and 89 GHz frequencies to water vapour in addition to
cloud, precipitation and the surface.

The bias of the distribution for the scenes $FG_{cloud}O_{clear}$ is positive for the low frequencies and negative for the high frequencies and ranges from -18 K to 50 K. For scenes $FG_{clear}O_{cloud}$ the bias is negative for the high frequencies and positive for the low frequencies and ranges from -45 K to 23 K. The sign of the bias changing between low and high frequencies is the result of the scattering effects which becomes stronger than the absorption/emission effects as the frequencies increase and are therefore in agreement with the scenes studied. For both scenes the biases of frequencies with horizontal polarization are degraded with respect to the vertical polarization. This problem is likely linked to the difficulty to simulate surface emissivity and the impossibility to simulate oriented particle emissivities for both polarizations within the RTTOV radiative transfer code.

The standard deviation for scenes $FG_{cloud}O_{clear}$ ranges from 7 K to 30 K and is generally higher for horizontally polarized channels. For scenes $FG_{clear}O_{cloudy}$ the same behavior is observed with a standard deviation ranging between 5 K and 30 K.

The following section examines the results of sensitivity studies on observation errors, channel selection and radiative properties used within the observation operator.
4.2 Sensitivity to observation error

a. Case study

The GMI instrument observed hurricane Maria in the Atlantic Ocean on September 18\textsuperscript{th}, 2017. A cloudy observation represented by the cross in Figure 2-f and located in the core of this hurricane is considered. Due to errors in the prediction of its track and its structure by AROME, the model state at the same location corresponds to an atmosphere with much less clouds and precipitation. The weights of a neighborhood defined by 625 profiles in a $250 \times 250$ km$^2$ box have been calculated and represented by black boxes for different values of the observation error $\sigma$: 1 K, 2 K, 5 K, 10 K and 20 K respectively in Figures 2-a, b, c, d and e. In this figure, the black boxes are filled according to the magnitude of their normalized weight. It appears that a large weight is given to a single profile for the 1, 2 and 5 K observation errors (Figures 2-a to 2-c) but when larger errors are considered, other profiles have a significant weight (Figures 2-d to 2-e).

For this particular case, the differences between the two estimators WA and ML are illustrated by the cumulative probability distribution, plotted as a function of relative humidity (Figure 3). The distributions are shown for a single level at 713 hPa (corresponding to AROME level 51). The impact of specifying a larger observation error can be noticed: the larger the observation error is, the more skewed the distribution is, towards dryer
values of relative humidity. The change in this distribution does not affect the ML estimator which always leads to a saturated atmosphere. But this affects the WA estimator which decreases towards less saturated amounts because of the long tailed distribution of probabilities.

Figures 4 and 5 show the retrieved atmospheric profiles with the WA and ML estimators, respectively for 1 K and 20 K errors. In both methods and for both errors, a low precipitating cloud layer is added to the first guess profile and the upper layer cloud is thickened with increased falling snow. As a result, the brightness temperatures simulated with the retrieved profiles are much closer to the observations in both cases. Like for relative humidity, the other atmospheric profiles are almost identical for the 1 K error for WA and ML, but strongly differ for 20 K. In particular, the retrieved cloud fraction for 20 K slowly decreases from 1 to 0 from 100 hPa to 1000 hPa. This retrieved of cloud fraction profile is not representative of AROME cloud fraction climatology which simulates frequently sharp binary profiles.

b. General results

In this section, five experiments with the observation errors $\sigma$ set to 1 K, 2 K, 5 K, 10 K and 20 K are intercompared over the 2-month period. Statistics between retrieved and observed brightness temperatures for both $FG_{\text{cloud}}O_{\text{clear}}$ and $FG_{\text{clear}}O_{\text{cloud}}$ scenes are shown Figure 6.
Regarding the correlation coefficients between retrieved and observed brightness temperatures, the \( F_{G_{\text{cloud}}}O_{\text{clear}} \) scenes lead to correlation values always higher than 0.7 (Figure 6-i-a). They are also higher for the smallest observation errors. In the case of 1 K and 2 K, the correlations are above 0.9 for all channels. For the \( F_{G_{\text{clear}}}O_{\text{cloud}} \) scenes, one can notice that the correlation coefficients are characterized by larger variations across channels compared to the \( F_{G_{\text{cloud}}}O_{\text{clear}} \) scenes. This makes sense because in the \( F_{G_{\text{cloud}}}O_{\text{clear}} \) case, no hydrometeor content is observed, so, in this case, the window channels are directly sensitive to the surface. In the case of \( F_{G_{\text{clear}}}O_{\text{cloud}} \), the expected retrieval is cloudy and precipitating and the correlations obtained with the different channels reflect the quality of the retrieval either for the liquid or the ice phase. Overall the correlations are always smaller for the low frequencies than for the higher ones. This indicates that the retrievals are of lower quality for liquid precipitation than for solid precipitation. This may be a result of the physics of the model, which may have difficulty in propagating realistic information about the liquid hydrometeor content when model profiles contain solid hydrometeors.

Regarding the variations of the correlation with the observation error specification the conclusions are similar to the \( F_{G_{\text{cloud}}}O_{\text{clear}} \) scenes: larger values are found with smaller observation errors. Overall, the correlation coeffi-
cient are significantly improved compared to the correlation coefficients of the first-guess (FG) (Figure 1).

Regarding the bias of retrieved brightness temperatures, the values are rather small for the $FG_{\text{cloud}}O_{\text{clear}}$ scenes (Figure 6-i-b), typically below 2 K. This means that the retrieval technique is well constrained for those scenes and does not create large biases. On the contrary for the $FG_{\text{clear}}O_{\text{cloud}}$ biases, Figure 6-ii-b shows that the bias can reach -40 K for $\sigma = 20 \text{ K}$. For smaller observation errors, like 1 K or 2 K, the biases are limited to values ranging between 5 K and -10 K. If these results are compared with the initial innovations of Figure 1, it can be seen that the biases are reduced regardless of the observation error used. The improvements are quite large for $FG_{\text{cloud}}O_{\text{clear}}$ scenes since there is almost no bias for each observation error. For $FG_{\text{clear}}O_{\text{cloud}}$ scenes, the improvements are less important but still significant.

Regarding the standard deviation of the brightness temperature differences for $FG_{\text{cloud}}O_{\text{clear}}$ scenes shown in Figure 6-i-c, it appears that the standard deviation increases with the observation error. The standard deviation never exceeds 5 K with an observation error of 1 K but can exceed 10 K for an observation error of 20 K. For the $FG_{\text{clear}}O_{\text{cloud}}$ scenes, the standard deviations have very similar values, except for the high frequencies and the 20 K observation error case.
The differences of results for observation errors can be further explained. Larger observation errors allow the retrievals to remain close to the FG values (i.e. far from observations) and thus to produce a weighted average of profiles with either too low or too much hydrometeor contents. The vicinity of observations being mostly composed of clear atmospheric profiles (see Table 4), the profile retrieved from the weighted average will be further influenced by clear profiles. The result will be to retrieve an atmospheric profile with a simulated brightness temperatures further clear than the observation. The example of Figure 5 illustrates such behavior. On the contrary, a low observation error will consider atmospheric profiles rather close to the observation and the retrieved profile will be less influenced by the difference in number between clear and cloudy scenes in its vicinity.

The retrieved profiles of relative humidity are compared to the FG profiles (analysis increments) in terms of bias and standard deviation statistics and displayed in Figure 7. The mean values for $FG_{\text{cloud}}O_{\text{clear}}$ scenes, Figure 7-i-a, are negative for all observation errors which means that the assimilation of retrieved profiles would tend to dry the model FG. The mean differences can reach up $-15\%$ between 600 and 800 hPa for the largest observation errors. For the $FG_{\text{clear}}O_{\text{cloud}}$ scenes, the mean values are always positive which means that the assimilation of the retrieved profiles will tend to moisten the
atmosphere. The smaller the observation error is, the larger the moistening will be: the 1 K and 2 K curves can reach up 12% of mean relative humidity FG departure. On the contrary, the mean values for the 20 K observation error are close to zero, which means that the assimilation of these retrievals will have very little effect on the analysis. Overall, these results indicate that a small observation error will enhance the FG moistening in case of cloudy observations.

When examining the standard deviations corresponding to the $FG_{cloudO_{clear}}$ scenes (Figure 7-i-b), the curves show almost no dependency with observation error and reach values up to 22% at 500 hPa. On the contrary, for the $FG_{clearO_{cloud}}$ scenes, there is a clear decrease of the standard deviations when larger observation errors are prescribed. Indeed, with an observation error of 20 K the maximum value is 15% whereas with a 1 K error the standard deviation can reach 23%.

From the above results, several conclusions can be drawn: small observation errors lead to the best fits between Bayesian retrievals and observations. Compared to the FG relative humidity profiles, retrievals with a small observation error either dry ($FG_{cloudO_{clear}}$) or moisten ($FG_{clearO_{cloud}}$) the atmosphere with a similar magnitude in average of +/- 10 to 15 % between 500 and 900 hPa.
On the contrary, larger observation errors tend to smooth the retrievals, with an overall degraded fit to observations. Differences of statistics between the WA and the ML estimators are not shown because (i) the ML estimator results do not vary with the observation error, (ii) the ML estimator results are almost identical to the WA result with a 1 K error.

4.3 Sensitivity to the channel selection

a. Case study

In this section, sensitivity studies to the channel selection have been undertaken. When considering the same case study as in section 4.1, the FG simulation (Figure 8-a) is very different from the observations (Figure 8-b). The hurricane structure of the FG appears more scattered with smaller scale structures than in the observations and the spiral bands are not well located. Three different Bayesian inversions with either one GMI frequency (18V or 183±7 GHz) or all GMI frequencies are performed, the corresponding simulated brightness temperatures over Hurricane Maria are respectively shown in Figures 8-c, 8-d and 8-e.

In the first case (Figure 8-c), the Hurricane structure retrieved with the
18.7V GHz GMI frequency is in good agreement with the observed one. Hence, the retrievals are well constrained for the low frequency 18V (Figure 8-c-i) but it is not the case for the 183±7 GHz frequency (Figure 8-c-ii) exhibiting a rather different brightness temperature structure. The spiral bands are characterized by too warm brightness temperatures whereas a strong underestimation is noticeable at the South of the core of the system. Regarding the experiment using the 183±7 GHz GMI frequency in the inversion, one finds the opposite results (Figure 8-d). The retrieved brightness temperatures have a structure in agreement with the observed ones at 183.31±7 GHz. On the contrary, when examining the retrieved structure at 18.7V GHz, the strong observed contrast between warm brightness temperatures in the cloudy regions and colder ones in clear sky regions is extremely blurred in the retrieval.

Finally, using all frequencies in the inversion leads to good results when compared to both frequencies (Figure 8-e).

From this case study, the retrievals appear to be well constrained in terms of brightness temperatures for the frequencies used within the Bayesian inversion. However, they lead to rather poor simulations of other frequencies. This means that this retrieval process can hardly infer information over the whole atmospheric profile. Nevertheless, it has been noticed that when us-
ing all channels in the inversion the retrieved profiles are better constrained. In the following we examine if these conclusions are also valid over a larger sample.

b. General results

A set of 11 experiments, which consists of introducing progressively the high to low frequencies in the Bayesian inversion were performed over the 2-month period. Statistics between retrieved and observed brightness temperatures for all experiments are shown Figure 9. The correlations for the $FG_{clear}O_{cloud}$ scenes (Figure 9-ii) are, like in the previous sections, worse than for the $FG_{cloud}O_{clear}$ scenes. Furthermore, they are low for the channels not used in the inversion and high for the other ones. For example, when using only high frequencies, the correlation coefficients are highly degraded for low frequencies with values down to 0.6 for the $FG_{cloud}O_{clear}$ scenes and down to 0.35 $FG_{clear}O_{cloud}$ scenes. When adding the low frequencies within the inversion the correlations increase progressively for both scenes. Only the 23.8 GHz frequency coefficients are relatively high for all experiments on Figure 9-ii-a but it was already the case for the initial innovations (Figure 1-a).
The biases of retrieved brightness temperatures are well constrained for the
\( FG_{\text{cloudOclear}} \) scenes (Figure 9-i-b): below 4 K. For the opposite weather
scenes (Figure 9-ii-b), the biases can reach -35 K for low frequencies when
only high frequencies are used in the inversion. Finally, by using all fre-
quencies, all biases are between -5 K and 5 K.

The standard deviations of retrieved brightness temperatures (Figure 9-
c) for the \( FG_{\text{cloudOclear}} \) scenes display low values with all frequencies and
increased ones for low frequency channels when only high frequencies are se-
lected. For the \( FG_{\text{clearOcloud}} \) scenes, the results are similar with larger stan-
dard deviation values. The lowest value is 6 K while for the \( FG_{\text{cloudOclear}} \) it
is only 2 K. Furthermore, the standard deviation of the 18H GHz frequency
channel reaches 31 K when only high frequencies are used.

In summary, the channels can be split in two sets having contrasted be-
haviors: the frequencies below 37 GHz and those above. The best results
for the high frequencies are obtained when using only high frequencies in
the inversion. When adding low frequencies, correlations for these channels
increase at the expense of slightly reduced correlations for high frequency
channels.
If one now considers the statistics of RH departures in terms of bias and standard deviations shown in Figure 10, results for the $FG_{cloud}O_{clear}$ scenes show almost no dependency with the number of channels considered in the inversion. The mean values are negative (leading to a model drying) and the standard deviation values can reach up to 22% between 300 and 600 hPa.

For the $FG_{clear}O_{cloud}$ scenes the results are significantly different (Figure 10-ii). The mean values are positive, so the retrieved profiles will tend to moisten the atmosphere. However, the maximum value of the mean gradually decreases with height as more channels are added to the inversion. Hence, with only the 183±3 and 183±7 GHz frequencies, the maximum values are located around 400 hPa and are respectively of 9% and 6%. Whereas the experiment using all channels reaches a maximum of 12% at 600 hPa. Moreover, when adding channels in the inversion, the mean relative humidity FG departure becomes progressively higher at several pressure levels. Thus, the addition of low frequency channels in the inversion allows relative humidity to be more modified closer to the surface.

Figure 11 shows, with the ML estimator, the correlation coefficients between retrieved and observed brightness temperatures for the experiment.
set adding channels from $183.31 \pm 7$ to $18.7$ GHz (Figure 11). The coefficients bring similar conclusions than with the WA estimator (Figure 9-a). However it can be noticed that the coefficients for the low frequency channel of experiments using only frequencies ranging from $183.31 \pm 7$ to $89$ GHz have smaller correlations with the ML estimator compared to the WA, for both category of scenes. The other statistics (biases and standard deviations) are very close to the WA estimator (not shown). Thus, the Bayesian inversion using the ML estimator seems less well constrained with a limited number of channels used than with the WA estimator.

A number of conclusions can be drawn from the above results. A large number of channels within the inversion allows to better constrain the retrievals on the vertical unlike the experiments with a reduced set (as done by Duruisseau et al. (2019) only with $183$ GHz channels). Indeed, the various channel help to provide information at pressure levels where they are sensitive to either solid or liquid hydrometeors. Furthermore, the FG will be moisten more homogeneously on the vertical with a full set of channels for the $FG_{clear,O_cloud}$ scenes. For the other scenes, the FG will be dried out in the same way: the fact that no hydrometeors are present in the observed scenes lead to rather similar retrieved atmospheric profiles. Finally the ML estimator has degraded performances with a limited set of channels.
4.4 Sensitivity to scattering properties of hydrometeors

Over the last decade, the radiative properties of frozen hydrometeors are a subject of research in the microwave assimilation community. Indeed, the scattering properties in the microwave depend strongly upon particle shape, density and size, which are extremely variable for solid hydrometeors within clouds. For the time being, the Météo-France data assimilation systems used with the RTTOV-SCATT model can only handle one particle shape and one particle size distribution per hydrometeor type, for all weather situations. Knowing the actual solid hydrometeor diversity, this represents a huge simplification of the atmosphere behavior (e.g. Schmitt et al., 2016). Hence, many studies have been undertaken and nowadays a number of databases provide radiative properties for a wide range of snow particles with various shapes and densities (e.g. Eriksson et al. 2018 ; Kneifel et al. 2020). In addition, the retrieval community is studying several methods to obtain information on hydrometeor diversity in the atmosphere from microwave observations and the assimilation community is just beginning to take into account this diversity in numerical models (e.g. Haddad et al. 2015). In this section, we examine the impact of the selection of radiative properties for hydrometeors onto the Bayesian inversion results. The Liu database (Liu,
2008) has been chosen because of its availability in the RTTOV-SCATT model and wide use in the community.

The FG and observed brightness temperature distributions over the 2-month period are displayed on Figure 12 for each particle of the Liu database. In this Figure, one can note that the longest tails of distributions toward cold brightness temperatures are associated with particle shapes having the strongest scattering efficiency. For frequencies 18V and 23V GHz (Figures 12-a and b), the distributions have a mode around 210 K and 250 K respectively, corresponding to clear sky scenes and are skewed towards warmer temperatures up to 280 K, corresponding to the microwave emission of liquid hydrometeors. The occurrences of simulated brightness temperatures between 210 K and 255 K for the 18V frequency and between 255 K and 280 K for the 23V frequency are smaller than the observed occurrences. These channels being sensitive to rain, thus seem to indicate a lack of liquid precipitation in the AROME model forecasts, which was already highlighted in Faure et al. (2020). Additionally, the distributions for each particle remain the same which is consistent with the fact that low frequencies are not sensitive to snow particles.

For the 36.64 GHz frequency (Figure 12-c) the distributions have a mode around 220 K with a significant positive skewness as noticed for the lower
frequencies. On the other hand, for a number of particles (the most efficient scatterers) the distributions are skewed towards colder temperatures down to 170 K. This behavior is the signature of scattering by solid particles which seems to overcome the emission signature in these cases.

For higher frequencies (89, 166 and 183 GHz) the distributions shown in Figures 12-d, e and f have a mode at warm temperatures (clear sky scenes) and a negative skewness associated with the scattering processes by frozen particles. Additionally, the observation distributions displayed in Figures 12-d, e and f appear to reasonably match the sector snowflake distributions as well as the bullet rosettes distributions. These results is consistent with the use of the sector snowflake particle as the reference in RTTOV-SCATT as suggested by Geer and Baordo (2014).

Furthermore, depending upon the choice of particle, the coldest simulated temperatures vary between 70 K and 150 K. Hence, it was chosen to rank particles according to their scattering efficiency to help the interpretation of the inversion results presented afterwards.

The Bayesian inversion has been run over the 2-month period with the 11 particle shapes. It was found that the retrieved brightness temperatures are characterized by highly similar statistics (not shown). The retrieval algorithm always finds a combination of FG profiles which does match the ob-
servations in the brightness temperature space. Nonetheless, the underlying retrieved atmospheric profiles are rather different. The mean distributions of retrieved hydrometeor profiles for the $FG_{clear}O_{cloud}$ scenes are displayed in Figure 13. It can be seen that the less efficient scattering particles tend to retrieve atmospheric profiles with the largest rain, snow and cloud ice water contents, as can be seen respectively on Figures 13-a, b and d. For example, the maximum of snow content means is 0.1 g/kg for one of the most efficient scattering particle shape, such as a block column, and at 0.65 g/kg for one of the least efficient scattering particle shape, such as the dendrite snowflake. For the mean retrieved cloud liquid water profiles shown in Figure 13-c, the behaviour is opposite. Indeed, the retrievals with the more efficient scattering particles are those containing the highest cloud liquid water content: up to 0.25 g/kg for the block column against 0.2 g/kg for dendrite snowflake. This suggests that the retrieval algorithm seeks to generate profiles with more rain, snow and cloud ice water but less cloud water for the less efficient scattering particles to match a cloudy observation. This might come from a lack of constrain on these microphysics species or a compensation effect which will need to be further investigated.

The statistics of relative humidity FG departures in terms of means and standard deviations are shown in Figure 14. The mean profiles for $FG_{cloud}O_{clear}$
scenes exhibit negative values leading to a drying of the atmosphere with variations in magnitude between 200 and 600 hPa. The most effective scattering particles lead to the largest atmospheric drying: for example, at 300 hPa, the mean values for the block column and the dendrite snowflake particles are respectively of -7% and -3%. For this category of scenes, one could have expected the particle shapes to have no effect on the retrieved profiles, given the fact that clear sky profiles do not or contain very little snow amounts. The fact that is not the case indicates that the filtering of meteorological scenes, based on the 36.64 GHz frequencies, does not fully discard all cloudy scenes, especially the non-precipitating ice clouds. Furthermore, the different meteorological weather scenes have been defined thanks to the brightness temperatures simulated with the sector snowflake particle. Another particle would have led to a different categorization but the filter based on the sector snowflake particle has been kept in order to compare an identical samples.

For the \( FG_{\text{clear\,O\,cloud}} \) scenes, the mean profiles are distributed towards positive values leading to an atmospheric moistening. Additionally two behaviors can be noticed:

- Between around 200 and 600 hPa, the less effective scattering particles, like the dendrite snowflake, tend to further moisten the atmosphere than other particles. This behavior is consistent with the results shown in Figure
the less effective scattering particles will retrieve a larger amount of
hydrometeors to reach the observed brightness temperatures.

- Between around 600 and 900 hPa, the opposite behavior can be seen. The
retrieval algorithm seeks to create a profile with further relative humidity
in the lower layer for the more effective scattering particles. This is a con-
sistent behavior with the mean distributions of retrieved cloud liquid water
profiles displayed in Figure 13.

In conclusion, the choice of frozen particles within the observation operator
has an important impact on retrieved relative humidity profiles. A most effi-
cient scatterer will tend to reduce the profile moistening in the $FG_{clear}O_{cloud}$
scenes. Opposite behaviour has been found over the mean distributions of
the cloud liquid water profiles and over the mean distributions of relative
humidity profiles in the lower layers. This will need further investigations
for a deeper understanding of this effect. Finally, because a 1 K observation
error was selected in the reference configuration, the results of statistics with
the ML estimator (not shown) are similar to those with the WA estimator.

5. Summary and Conclusions

A number of sensitivity studies has been performed on various specifications
of the ’1D-Bay+3D/4D-Var’ method over a 2-month period in 2017 with the
convective scale model AROME and GMI brightness temperatures from 18 to 183 GHz. The following aspects have been examined (i) the observation errors, (ii) the channel selection, and (iii) the scattering radiative properties of frozen hydrometeors in the observation operator. On top of assessing the impact of these specifications, two estimators have been compared: the mean of likelihood distribution (so called weighted average: WA) and the maximum value of this distribution (so called maximum likelihood: ML).

Observation errors ranging from 1 K to 20 K have been tested for the inversion with the WA estimator. Large errors tend to smooth and dry the retrieved profiles, as well as producing less realistic hydrometeor profiles. In constrast, small observation error tend to retrieve profiles more consistent with observed brightness temperatures. Furthermore, with small errors, the retrieved profiles are similar with both estimators, the ML one being invariant to this parameter when using a covariance error matrix with single value along its diagonal. In a future study, dedicated diagnostics could be applied to estimate optimal parameters for the assimilation (e.g. Desroziers et al., 2005).

Eleven experiments have been carried out regarding the channel selection. Compared to previous studies (Guerbette et al. 2016 and Duruisseau et
al. 2018), it was shown that the inversion does work with a larger set of channels. Adding low frequency channels to the inversion brings information on the low atmospheric layers of the atmosphere and leads to good quality retrievals as shown in the observation space. Moreover, the results highlight the usefulness of two groups of frequencies, the lower ones (from 18V to 36H GHz) and the higher ones (from 89V to 183±7 GHz). Using low and high frequencies allows to constrain the retrievals below and above the freezing level respectively.

The inversion experiments were performed for each frozen hydrometeor shape from the Liu’s database of scattering properties (Liu, 2008). The conclusions drawn from this study are that the simulated brightness temperatures of the retrieved profiles remain consistent with observed brightness temperatures with any of the scattering properties selected. However, depending on the choice of particle shape for snowfall representation, the retrieved atmospheric profiles can vary significantly at high levels (moister profiles for less efficient scatterers). Indeed, a weakly scattering particle leads to a retrieval with larger hydrometeor and relative humidity contents. The opposite behavior was also found for the cloud liquid water content and the relative humidity content at low levels and need further investigations.

Finally, the ML and the WA estimators show differences over two cases:
ML does not depend on the observation error which is one strength of this estimator. However the fit to observations is degraded with ML when using a reduced number of channels.

This study brings a number of perspectives for testing the GMI retrieval assimilation as well as retrievals from other instruments of the GPM constellation.

First, the sensitivity study on the observation error highlighted the benefit of considering low values. Then, depending on the instrument frequencies selected in the Bayesian inversion, the resulting relative humidity profiles should be filtered out vertically. This will allow the assimilation of relative humidity in the 3D-Var or in the 4D-Var only at relevant pressure levels. To assess the quality of the retrievals, an independent validation would be to compare them with reflectivity profiles from the Dual frequency Precipitation Radar onboard the GPM Core satellite. Then, the Bayesian inversion could be enhanced by additional channels. In particular, the use of temperature sounding channels will be investigated in the future to better constrain the retrievals. The MicroMas instruments onboard Time-Resolved Observations of Precipitation structure and storm Intensity with a Constellation of Smallsats (TROPICS) could be used to conduct such study (Blackwell et al. 2012).

The sensitivity study on the radiative properties highlighted that, as in
the direct assimilation methods, it is important to optimize the radiative properties within the observation operator. One interesting feature of the Bayesian inversion over the direct assimilation of brightness temperatures is that it is possible to use several different radiative properties within the inversion thus extending the database of profiles. This possibility will be tested in a future study.

Finally, the maximum likelihood estimator shows both advantages and disadvantages over the weighted mean estimator. Therefore, further comparisons will be performed and evaluated within assimilation experiments. One interesting feature of the Bayesian inversion is that it is possible to use different radiative properties of hydrometeors within the inversion by extending the database of simulated profiles. This option can be used both with the WA and ML estimators. This will be the subject of future research to further explore how the variability of hydrometeor shapes and distributions observed in nature can be represented within a data assimilation system.

Acknowledgements

This research is funded by Météo-France and Région Occitanie (PhD grant for Marylis Barreyat). The authors acknowledge the Centre National d’Études Spatiales (CNES) for the financial support of this scientific research activity.
part of the Infrarouge, Micro-Ondes et Transfert radiatif ensembliste pour la prévision des Extrêmes de Précipitations (IMOTEP) project. The two anonymous reviewers are acknowledged for their useful comments which helped improving the manuscript.

References


Bauer, P., E. Moreau, F. Chevallier, and U. O’Keeffe, 2006c: Multiple-scattering microwave radiative transfer for data assimilation appli-


Eriksson, P., R. Ekelund, J. Mendrok, M. Brath, O. Lemke, and S. A. Buehler, 2018: A general database of hydrometeor single scatter-
ing properties at microwave and sub-millimetre wavelengths. Earth System Science Data, 10(3), 1301–1326.


Field, P. R., A. J. Heymsfield, and A. Bansemer, 2007: Snow size distribution parameterization for midlatitude and tropical ice clouds. Journal of the atmospheric Sciences, 64(12), 4346–4365.


Guerbette, J., J.-F. Mahfouf, and M. Plu, 2016: Towards the assimilation of all-sky microwave radiances from the SAPHIR humidity sounder in a limited area NWP model over tropical regions. Tellus A: Dynamic Meteorology and Oceanography, 68(1), 28620.

for the nonlinear dependence of scattering microwave observations
of precipitation. *Journal of Geophysical Research: Atmospheres,*
**120**(11), 5548–5563.

Hou, A. Y., R. K. Kakar, S. Neeck, A. A. Azarbarzin, C. D. Kum-
merow, M. Kojima, R. Oki, K. Nakamura, and T. Iguchi, 2014: The
Meteorological Society,* **95**(5), 701–722.

Hou, A. Y., and S. Q. Zhang, 2007: Assimilation of precipitation informa-
tion using column model physics as a weak constraint. *Journal of
the atmospheric sciences,* **64**(11), 3865–3878.

Hou, A. Y., S. Q. Zhang, and O. Reale, 2004: Variational continuous assim-
ilation of tmi and ssm/i rain rates: Impact on geos-3 hurricane anal-

Ikuta, Y., 2016: Data assimilation using GPM/DPR at JMA. *CAS/JSC
WGNE Research Activities in Atmospheric and Oceanic Modelling,*
**46**, 01–11.

Ikuta, Y., and Y. Honda, 2011: Development of 1D+ 4DVAR data assim-
ilation of radar reflectivity in JNoVA. *CAS/JSC WGNE Res. Activ.
Atmos. Oceanic Modell,* **41**, 01–09.


Wattrelot, E., O. Caumont, and J.-F. Mahfouf, 2014: Operational implementation of the 1D+3D-Var assimilation method of radar reflectiv-
List of Figures

1. (a): Correlation coefficients, (b) bias and (c) standard deviations between the observations and the FG, uncorrected of the clear sky bias. Column (i) represents the results for experiment $FG_{cloud}O_{clear}$ and column (ii) those for $FG_{clear}O_{cloud}$. Stars indicate results which are not significative at 95% level. Statistics computed over the two-month period from September to October 2017.

2. a, b, c, d, e - FG brightness temperature simulations of the Maria hurricane reaching the West Indies on September 18th, 2017 for the 18H GHz frequency. Inversion weights are represented as the filling fraction of the black boxes for an observation error equals to 1 K, 2 K, 5 K, 10 K and 20 K, respectively figures a, b, c, d, and e. f - GMI observations of the Hurricane with the 18H GHz frequency. The selected observations is represented by the black cross. On this figure, the black boxes are filled according to the magnitude of their normalized weight.

3. (a), (b), (c), (d), and (e): Cumulated histograms of the weights from the Bayesian inversion for the case study by bins of relative humidity of 5% at 713 hPa for an observation error of respectively 1 K, 2 K, 5 K, 10 K and 20 K. The gray, blue and red arrows respectively indicate the values of the FG relative humidity and the WA and the ML retrieved relative humidity. (a) to (c) the blue and red arrows are identical.

4. (a), (b), (c), (d), (e), and (f): Case study atmospheric profiles of the relative humidity, cloud fraction, rain, snow, cloud liquid water and cloud ice water for the FG in gray, the WA retrieval in blue and the ML retrieval in dotted red with an observation error equal to 1 K. (g): GMI brightness temperatures are represented by a black square for the observations, by a gray circle for the FG, by a blue triangle for the WA retrievals and by a red triangle for the ML retrievals with the same observation error.

5. Same as Figure 4, with an observation error of 20 K.
(a): Correlation coefficient, (b) bias and standard deviation distributions of GMI brightness temperature differences between the observations and the WA experiments, uncorrected of the clear sky bias, for observation errors equal to 1 K, 2 K, 5 K, 10 K and 20 K. Column (i) represents the results for experiment $FG_{cloud}O_{clear}$ and column (ii) those for $FG_{clear}O_{cloud}$. Statistics are computed over the two-month period from September to October 2017.

(a) and (b): Mean and standard deviation distributions of the relative humidity differences between the WA experiments and the FG for observation errors equal to 1 K, 2 K, 5 K, 10 K and 20 K. Column (i) represents the results for the experience $FG_{cloud}O_{clear}$ and column (ii) those for $FG_{clear}O_{cloud}$. Statistics are computed over the two-month period from September to October 2017.

GMI brightness temperatures of Hurricane Maria in September 18th, 2017 (a): FG simulations, (b): observations, (c): WA retrieved simulations with the use of only the 18.7V GHz frequency in the inversion, (d): WA retrieved brightness temperature simulations with the use of only the 183$\pm$7 GHz frequency in the inversion and (e): WA retrieved simulations with the use of all GMI frequencies in the inversion, for channel 18.7V GHz column (i) and for channel 183$\pm$7 GHz column (ii).

(a): Correlation coefficient, (b) bias and (c) standard deviation distributions of GMI brightness temperature differences between the WA experiments and the observations, uncorrected of the clear sky bias, adding channels from 183$\pm$7 GHz to 18.7V GHz. Column (i) represents the results for experiment $FG_{cloud}O_{clear}$ and column (ii) those for $FG_{clear}O_{cloud}$. Each color corresponds to an experiment with adding channels, the curves are also thicker with the increase in the number of channels. Statistics are computed over the two-month period from September to October 2017.
(a) and (b): Mean and standard deviation distributions of the relative humidity differences between the WA experiments and the FG adding channels from 183±7 GHz to 18.7V GHz. Column (i) represents the results for experiment $FG_{cloud}O_{clear}$ and column (ii) those for $FG_{clear}O_{cloud}$. As in Figure 9, curves are drawn thicker with the number of channels increasing. Statistics are computed over the two-month period from September to October 2017.

Correlation coefficient between the observations (GMI brightness temperatures) and the ML experiments adding channels from 183±7 GHz to 18.7V GHz. Column (i) represents the results for experiment $FG_{cloud}O_{clear}$ and column (ii) those for $FG_{clear}O_{cloud}$. As in Figure 9, curves are drawn thicker with the number of channels increasing. Statistics are computed over the two-month period from September to October 2017.

(a) to (f): Distributions of GMI FG brightness temperatures for each particle at frequency 18.7V GHz (a), 23.8V GHz (b), 36.64V GHz (c), 89V GHz (d), 166V GHz (e) and 183.31±7 GHz (f). Statistics computed over the two-month period from September to October 2017.

Mean retrieved profiles of (a) rain, (b) snow, (c) cloud liquid water and (d) cloud ice water for the various particles in the Liu table (2008) for $FG_{clear}O_{cloud}$ scenes. Statistics are computed over the two-month period from September to October 2017.

Mean distributions of relative humidity differences between the FG and the WA experiments for particles from the Liu database (2008). Column (i) represents the results for the experiment $FG_{cloud}O_{clear}$ and column (ii) those for $FG_{clear}O_{cloud}$. Statistics are computed over the two-month period from September to October 2017.
Fig. 1: (a): Correlation coefficients, (b) bias and (c) standard deviation distributions of GMI brightness temperature differences between the observations and the FG, uncorrected of the clear sky bias. Column (i) represents the results for experiment $FG_{cloud\ O_{clear}}$ and column (ii) those for $FG_{clear\ O_{cloud}}$. Stars indicate results which are not significative at 95% level. Statistics computed over the two-month period from september to october 2017.
Fig. 2: a, b, c, d, e - FG brightness temperature simulations of the Maria hurricane reaching the West Indies on September 18th, 2017 for the 18H GHz frequency. Inversion weights are represented as the filling fraction of the black boxes for an observation error equals to 1 K, 2 K, 5 K, 10 K and 20 K, respectively figures a, b, c, d, and e. f - GMI observations of the Hurricane with the 18H GHz frequency. The selected observations is represented by the black cross. On this figure, the black boxes are filled according to the magnitude of their normalized weight.
Fig. 3: (a), (b), (c), (d), and (e): Cumulated histograms of the weights from the Bayesian inversion for the case study by bins of relative humidity of 5% at 713 hPa for an observation error of respectively 1 K, 2 K, 5 K, 10 K and 20 K. The gray, blue and red arrows respectively indicate the values of the FG relative humidity and the WA and the ML retrieved relative humidity. (a) to (c) the blue and red arrows are identical.
Fig. 4: (a), (b), (c), (d), (e), and (f): Case study atmospheric profiles of the relative humidity, cloud fraction, rain, snow, cloud liquid water and cloud ice water for the FG in gray, the WA retrieval in blue and the ML retrieval in dotted red with an observation error equal to 1 K. (g): GMI brightness temperatures are represented by a black square for the observations, by a gray circle for the FG, by a blue triangle for the WA retrievals and by a red triangle for the ML retrievals with the same observation error.
Fig. 5: Same as Figure 4, with an observation error of 20 K.
Fig. 6: (a): Correlation coefficient, (b) bias and standard deviation distributions of GMI brightness temperature differences between the observations and the WA experiments, uncorrected of the clear sky bias, for observation errors equal to 1 K, 2 K, 5 K, 10 K and 20 K. Column (i) represents the results for experiment $FG_{cloud}O_{clear}$ and column (ii) those for $FG_{clear}O_{cloud}$. Statistics are computed over the two-month period from September to October 2017.
Fig. 7: (a) and (b): Mean and standard deviation distributions of the relative humidity differences between the WA experiments and the FG for observation errors equal to 1 K, 2 K, 5 K, 10 K and 20 K. Column (i) represents the results for the experience $FG_{\text{cloud}}O_{\text{clear}}$ and column (ii) those for $FG_{\text{clear}}O_{\text{cloud}}$. Statistics are computed over the two-month period from September to October 2017.
Fig. 8: GMI brightness temperatures of Hurricane Maria in September 18th, 2017 (a): FG simulations, (b): observations, (c): WA retrieved simulations with the use of only the 18.7V GHz frequency in the inversion, (d): WA retrieved brightness temperature simulations with the use of only the 183±7 GHz frequency in the inversion and (e): WA retrieved simulations with the use of all GMI frequencies in the inversion, for channel 18.7V GHz column (i) and for channel 183±7 GHz column (ii).
Fig. 9: (a): Correlation coefficient, (b) bias and (c) standard deviation distributions of GMI brightness temperature differences between the WA experiments and the observations, uncorrected of the clear sky bias, adding channels from 183±7 GHz to 18.7V GHz. Column (i) represents the results for experiment $FG_{\text{cloud}}O_{\text{clear}}$ and column (ii) those for $FG_{\text{clear}}O_{\text{cloud}}$. Each color corresponds to an experiment with adding channels, the curves are also thicker with the increase in the number of channels. Statistics are computed over the two-month period from september to october 2017.
Fig. 10: (a) and (b): Mean and standard deviation distributions of the relative humidity differences between the WA experiments and the FG adding channels from \(183\pm7\) GHz to 18.7V GHz. Column (i) represents the results for experiment \(FG_{\text{cloud}}O_{\text{clear}}\) and column (ii) those for \(FG_{\text{clear}}O_{\text{cloud}}\). As in Figure 9, curves are drawn thicker with the number of channels increasing. Statistics are computed over the two-month period from September to October 2017.
Fig. 11: Correlation coefficient between the observations (GMI brightness temperatures) and the ML experiments adding channels from 183±7 GHz to 18.7V GHz. Column (i) represents the results for experiment $FG_{\text{cloud}}O_{\text{clear}}$ and column (ii) those for $FG_{\text{clear}}O_{\text{cloud}}$. As in Figure 9, curves are drawn thicker with the number of channels increasing. Statistics are computed over the two-month period from September to October 2017.
Fig. 12: (a) to (f): Distributions of GMI FG brightness temperatures for each particle at frequency 18.7V GHz (a), 23.8V GHz (b), 36.64V GHz (c), 89V GHz (d), 166V GHz (e) and 183.31±7 GHz (f). Statistics computed over the two-month period from September to October 2017.
Fig. 13: Mean retrieved profiles of (a) rain, (b) snow, (c) cloud liquid water and (d) cloud ice water for the various particles in the Liu table (2008) for $F_{G \text{clear}O_{\text{cloud}}}$ scenes. Statistics are computed over the two-month period from September to October 2017.
Fig. 14: Mean distributions of relative humidity differences between the FG and the WA experiments for particles from the Liu database (2008). Column (i) represents the results for the experiment $FG_{\text{cloud}}O_{\text{clear}}$ and column (ii) those for $FG_{\text{clear}}O_{\text{cloud}}$. Statistics are computed over the two-month period from September to October 2017.
List of Tables

1 List of GMI channels with several instrument characteristics relevant for the study including the radiometric accuracy and the selected superobbing resolution (Skofronick-Jackson et al. 2017). IFOV denotes the Instantaneous Fields of View and Ne∆T the Noise Equivalent Delta Temperature. . . . . . . . 70

2 RTTOV-SCATT configurations selected for simulating GMI within the AROME model. . . . . . . . . . . . . . . . . . . . . . . . 71

3 Parameter values for the experiments and the reference. . . 72

4 Number of cases of the meteorological scenes comparing FG profiles and GMI observations over a two-month period based over the P37 index and brightness temperatures simulated with the sector snowflake particle in the Liu database. $FG_{clear}$ and $FG_{cloud}$ denote respectively a clear profile and a cloudy profile in the FG. $O_{clear}$ and $O_{cloud}$ denote respectively a clear and a cloudy GMI observation. . . . . . . . . . . . . . . . . . . . . . . . 73
<table>
<thead>
<tr>
<th>ID</th>
<th>Central Frequency (GHz)</th>
<th>Bandwidth (MHz)</th>
<th>Polarization</th>
<th>Ne∆T (K)</th>
<th>Viewing Angle (°)</th>
<th>IFOV (km)</th>
<th>Superobbed Resolution (°)</th>
<th>Clear Sky Bias of FG departures (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.65</td>
<td>100</td>
<td>V</td>
<td>0.96</td>
<td>52.8</td>
<td>19×32</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>2</td>
<td>10.65</td>
<td>100</td>
<td>H</td>
<td>0.96</td>
<td>52.8</td>
<td>19×32</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>3</td>
<td>18.7</td>
<td>200</td>
<td>V</td>
<td>0.84</td>
<td>52.8</td>
<td>11×18</td>
<td>0.1 × 0.1</td>
<td>-0.65</td>
</tr>
<tr>
<td>4</td>
<td>18.7</td>
<td>200</td>
<td>H</td>
<td>0.84</td>
<td>52.8</td>
<td>11×18</td>
<td>0.1 × 0.1</td>
<td>0.09</td>
</tr>
<tr>
<td>5</td>
<td>23.8</td>
<td>400</td>
<td>V</td>
<td>1.05</td>
<td>52.8</td>
<td>9.2 × 15</td>
<td>0.1 × 0.1</td>
<td>-0.92</td>
</tr>
<tr>
<td>6</td>
<td>36.64</td>
<td>1000</td>
<td>V</td>
<td>0.65</td>
<td>52.8</td>
<td>8.6 × 14</td>
<td>0.1 × 0.1</td>
<td>0.02</td>
</tr>
<tr>
<td>7</td>
<td>36.64</td>
<td>1000</td>
<td>H</td>
<td>0.65</td>
<td>52.8</td>
<td>8.6 × 14</td>
<td>0.1 × 0.1</td>
<td>-0.96</td>
</tr>
<tr>
<td>8</td>
<td>89.0</td>
<td>6000</td>
<td>V</td>
<td>0.57</td>
<td>52.8</td>
<td>4.4 × 7.2</td>
<td>0.1 × 0.1</td>
<td>0.33</td>
</tr>
<tr>
<td>9</td>
<td>89.0</td>
<td>6000</td>
<td>H</td>
<td>0.57</td>
<td>52.8</td>
<td>4.4 × 7.2</td>
<td>0.1 × 0.1</td>
<td>-0.29</td>
</tr>
<tr>
<td>10</td>
<td>166</td>
<td>4000</td>
<td>V</td>
<td>1.5</td>
<td>49.1</td>
<td>4.4 × 7.2</td>
<td>0.1 × 0.1</td>
<td>-0.4</td>
</tr>
<tr>
<td>11</td>
<td>166</td>
<td>4000</td>
<td>H</td>
<td>1.5</td>
<td>49.1</td>
<td>4.4 × 7.2</td>
<td>0.1 × 0.1</td>
<td>-0.2</td>
</tr>
<tr>
<td>12</td>
<td>183.3±3</td>
<td>2000</td>
<td>V</td>
<td>1.5</td>
<td>49.1</td>
<td>4.4 × 7.2</td>
<td>0.1 × 0.1</td>
<td>0.39</td>
</tr>
<tr>
<td>13</td>
<td>183.3±7</td>
<td>2000</td>
<td>V</td>
<td>1.5</td>
<td>49.1</td>
<td>4.4 × 7.2</td>
<td>0.1 × 0.1</td>
<td>-0.66</td>
</tr>
</tbody>
</table>

Table 1: List of GMI channels with several instrument characteristics relevant for the study including the radiometric accuracy and the selected superobbing resolution (Skofronick-Jackson et al. 2017). IFOV denotes the Instateneous Fields of View and Ne∆T the Noise Equivalent Delta Temperature.
<table>
<thead>
<tr>
<th>Hydrometeors</th>
<th>Particle size distribution</th>
<th>Single emission/absorption/scattering</th>
<th>Radiative properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td>Marshall-Palmer</td>
<td></td>
<td>Mie sphere</td>
</tr>
<tr>
<td>Snow</td>
<td>Field et al. (2007), Tropical regions</td>
<td>Discrete Dipole Approximations (DDA) 11 different shapes from Liu (2008)</td>
<td></td>
</tr>
<tr>
<td>Cloud liquid water</td>
<td>Modified Gamma</td>
<td></td>
<td>Mie sphere</td>
</tr>
<tr>
<td>Cloud ice water</td>
<td>Modified Gamma</td>
<td></td>
<td>Mie sphere</td>
</tr>
</tbody>
</table>

Table 2: RTTOV-SCATT configurations selected for simulating GMI within the AROME model.
<table>
<thead>
<tr>
<th>Degrees of freedom</th>
<th>Reference</th>
<th>Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>The observation error</td>
<td>1 K</td>
<td>2 K, 5 K, 10 K, 20 K</td>
</tr>
<tr>
<td>GMI channels</td>
<td>All channels from 18.7 to 183.3±7</td>
<td>Adding channels from 183.3±7 to 18.7 GHz</td>
</tr>
<tr>
<td>Radiative properties for snow particle</td>
<td>Sector Snowflake</td>
<td>Long Column, Short Column, Block Column, Thick Plate, Thin Plate, Rosette 3-bullet, Rosette 4-bullet, Rosette 5-bullet, Rosette 6-bullet, Dendrite Snowflake</td>
</tr>
</tbody>
</table>

Table 3: Parameter values for the experiments and the reference.
Table 4: Number of cases of the meteorological scenes comparing FG profiles and GMI observations over a two-month period based over the P37 index and brightness temperatures simulated with the sector snowflake particle in the Liu database. $FG_{\text{clear}}$ and $FG_{\text{cloud}}$ denote respectively a clear profile and a cloudy profile in the FG. $O_{\text{clear}}$ and $O_{\text{cloud}}$ denote respectively a clear and a cloudy GMI observation.