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H_∞ Filtering for Bias Correction in Post-Processing of Numerical Weather Prediction

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Abstract

In this paper, we propose an H-infinity (H_∞) filtering approach for the prediction of bias in post-processing of model outputs and past measurements. This method adopts minimax strategy that is a solution for zero-sum games. The proposed H_∞ filtering approach minimizes maximum possible errors whereas a recently common approach that adopts the Kalman filtering (KF) minimizes the mean square errors. The proposed approach does not need the information of noise statistics unlike the method based on the KF, while training process is required. We show that the proposed approach outperforms the method based on the KF in experiments by applying real weather data in Korea.

Keywords Kalman filtering; H-infinity filtering; model post-processing; numerical weather prediction

1. Introduction

Weather forecasting is important and is closely related to activities of our daily life. As recent global interest, natural resources such as wind power and photovoltaic energy are highly related with renewable green energy. Particularly, irradiance forecast is an essential factor for photovoltaic energy management (Pelland, Galanis & Kallos, 2013).

Weather forecasting technology is mainly relying on super computer systems such as “numerical weather prediction (NWP)” model systems. These systems model and predict various meteorological variables based on past massive database with enormous computational cost. However, we cannot avoid the biases between real measurements and predicted values of the weather. It is known that the best way to obtain objective forecasts of local weather parameters is to use statistical methods to complement raw outputs of the NWP models (Klein & Glahn, 1974). Therefore, we post-process past-predictions and past-measurements to adjust the predicted biases. There are various methods for this model post-processing such as rank histogram recalibration (Hamill & Colucci, 1998), Bayesian processor of output/forecast (Krzysztofowicz & Maranzano, 2006), ensemble model output statistics (Scheuerer & Büermann, 2014), neural networks (Lauret, Diagne & David, 2014), etc.

The bias can be modeled by a polynomial function with respect to a predicted meteorological variable such as temperature or irradiance, etc. If we obtain coefficients of the polynomial function, then the bias can be estimated that is used for correcting predicted meteorological variable of NWP. An approach based on Kalman filtering (KF) for the bias correction in wind speed and temperature forecast was proposed in (Louka, Galanis, Siebert, Kariniotakis, Katsafados, Pytharoulis & Kallos, 2008)(Galanis, Louka, Katsafados, Pytharoulis & Kallos, 2006). For irradiance forecast, the KF was adopted in (Pelland, Galanis & Kallos, 2013) to outperform conventional approaches.

In this paper, we propose an H-infinity (H_∞) filtering approach for mitigating the biases of NWP in post-processing to better predict horizontal irradiance. While the KF minimizes mean square errors (MSE)s of estimates, H_∞ filter minimizes the worst possible errors (Shen & Deng, 1997)(Lim, 2014)(Simon, 2006). Therefore, in H_∞ filtering, the strategy to solve the problem is the same as in the minimax estimator which is a solution for zero-sum games. Furthermore, although we need to train the H_∞ filter in accordance with the variances of the state and the measurement noises, true values of the variances are not needed for implementations of H_∞ filtering.

Therefore, we do not need to know the noise statistics of the problem unlike the method based on the KF. Noise statistics usually mean the "mean" and "variance or covariance" of the noise. The KF is the optimal approach in terms of minimum MSE criterion only when the state and observation equations are linear functions with respect to the state on condition that the noises for two equations are Gaussian whose expectations and the variances are known. However, in practical problems such as weather forecast, we may not be able to know the noise statistics exactly leaving the Gaussian noise part aside. Therefore, it is very difficult to obtain optimal results by the KF in practice. For this reason, we take H_∞ filtering that adopts the strategy of minimizing maximum errors rather than minimizing MSE. Particularly, in this weather prediction problem, it is highly demanded to reduce large errors, which is the strategy of H_∞ filtering. In this paper, we show outperforming results via the proposed H_∞ filtering approach over the KF. This approach can be applied for correcting the bias of any predicted meteorological variables by NWP. For notations, square matrices, vectors, and scalars are denoted by bold uppercase, bold lowercase, and lowercase letters, respectively.

82 2. Problem Formulation

NWP systems predict values of meteorological variables based on past massive data. If we process prediction bias in the past, we can improve the prediction accuracy based on mitigated bias. The first step of this post-processing is to model the bias that is the difference between the forecasted and the measured values for a certain time (which is determined as a target time of a day) on the day k, defined as

$$\mathcal{B}_k = m_{k,f} - m_{k,r},\ k = 1, \dots, \tag{1}$$

where $m_{k,f}$ is a predicted value of a meteorological variable (e.g. temperature, wind speed, irradiance), i.e. the direct output of NWP systems, and $m_{k,r}$ is the real measurement of the corresponding variable at the target time of a day k. It needs to be noted that k is not the hour index but the day index. We wish to minimize the difference of a forecasted value $m_{k,f}$ from its measured value $m_{k,r}$. We model the bias $\mathcal{B}_k$ by y_k that is represented by a nth degree (defining $\eta = n + 1$) linear polynomial of the forecasted value with additive noise w_k given by,

$$y_k = x_{k,0} + x_{k,1} \cdot m^1_{k,f} + x_{k,2} \cdot m^2_{k,f} + \cdots + x_{k,n} \cdot m^n_{k,f} + w_k. \tag{2}$$

Using a column vector of indeterminates $\boldsymbol{x}_k$ (η elements)

$$\boldsymbol{x}_k = [x_{k,0}\ \ x_{k,1}\ \ x_{k,2}\ \ x_{k,3}\ \ \dots\ \ x_{k,n}]^\top, \tag{3}$$

we can also describe the bias $y_k = \boldsymbol{g}_k \boldsymbol{x}_k + w_k$, where a vector of η elements

$$\boldsymbol{g}_k = \begin{bmatrix} 1 & m_{k,f}^1 & m_{k,f}^2 & \ldots & m_{k,f}^n \end{bmatrix}. \tag{4}$$

We need to obtain $\boldsymbol{x}_k$ sequentially to solve the problem.

The problem described above can be solved by the KF which was recently proposed as a bias correction approach. In this problem, the KF estimates the polynomial coefficients $\boldsymbol{x}_k$ sequentially, and the final estimate can be employed for the bias of the following day. The system model for the KF can be described in the following "dynamic state system" and "the measurement equation,"

$$\boldsymbol{x}_k = \boldsymbol{F}\boldsymbol{x}_{k-1} + \boldsymbol{u}_k, \tag{5}$$

$$y_k = \boldsymbol{g}_k \boldsymbol{x}_k + w_k, \tag{6}$$

where $\boldsymbol{F}$ and $\boldsymbol{g}_k$ are matrices rather than functions in this case, $\boldsymbol{u}_k$ is the $\eta \times 1$ process noise vector of the state system, and w_k is the measurement noise, respectively. It is well known that the KF is the optimal solution when Eqs. (5) and (6) are linear functions with respect to $\boldsymbol{x}$, and $\boldsymbol{u}$ and w_k are Gaussian distributed on condition that all equations and noise statistics are known. The steps of the Kalman algorithm are provided as follows.

$$\text{Prediction} \quad \bar{\boldsymbol{x}}_k = \boldsymbol{F}\hat{\boldsymbol{x}}_{k-1}, \tag{7}$$

$$\text{Predicted covariance} \quad \bar{\boldsymbol{P}}_k = \boldsymbol{F}\boldsymbol{P}_{k-1}\boldsymbol{F}^\top + \boldsymbol{Q}_{u,k}, \tag{8}$$

$$\text{Kalman gain} \quad \mathcal{K}_k = \bar{\boldsymbol{P}}_k\boldsymbol{g}_k^\top \left(\boldsymbol{g}_k\bar{\boldsymbol{P}}_k\boldsymbol{g}_k^\top + q_{w,k}\right)^{-1}, \tag{9}$$

$$\text{Estimate} \quad \hat{\boldsymbol{x}}_k = \bar{\boldsymbol{x}}_k + \mathcal{K}_k\left(y_k - \boldsymbol{g}_k\bar{\boldsymbol{x}}_k\right), \tag{10}$$

$$\text{Covariance} \quad \boldsymbol{P}_k = \bar{\boldsymbol{P}}_k - \mathcal{K}_k\boldsymbol{g}_k\bar{\boldsymbol{P}}_k, \tag{11}$$

where $\boldsymbol{Q}_{u,k}$ and $q_{w,k}$ are the covariance of $\boldsymbol{u}_k$ and the variance of w_k, respectively. Before we solve the problem, we first determine a horizon (between 0 and 48 hours) and the target time of a day to predict for. The horizon means the length of time ahead of the target time of a day we want to predict for. Although in meteorology, "period" is commonly used instead of "horizon," we employ the term of horizon that is also used in the literature including the benchmark paper (Pelland, Galanis & Kallos, 2013). To correct a bias, data of previous 30 to 90 days are used, which is defined as the window size K. For example, if we want to predict a weather variable at 10 a.m. on May 1, 2014 with the horizon 1 hour ahead, we need real observations at 10 a.m. and predictions at 9 a.m. of the days, at least, from April 1 to April 30. In this case, K equals 30. Therefore, we obtain the estimate $\boldsymbol{x}_K$ by the KF, and approximate the bias at $K+1$ by $y_K = \boldsymbol{g}_K\boldsymbol{x}_K$ to obtain adjusted forecast $m'_{K+1,f} = m_{K+1,f} - y_K$. In our problem, $\boldsymbol{F}$ is an identity matrix, then Eq. (5) becomes $\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \boldsymbol{u}_k$. By using a filter such as the Kalman filter, we can correct an NWP output as described in Fig. 1. Fig. 1

3. Proposed H_∞ Filtering Approach

Based on the system model, Eqs. (5) and (6), we describe the proposed H_∞ filtering approach for correcting the bias between NWP predictions and real measurements. The H_∞ filter is employed in the "Filter" block in the post-processing of Fig. 1.

H_∞ filtering applications are designed to ensure the H_∞ norm is less than a predetermined bound based on noisy signals and resulting estimation errors, which is defined in Eq. (12) below. In this approach, the noise source can be arbitrary while it is bounded by a certain value, and the possible worst signal with error is minimized, which is similar to minimax solution for zero-sum games. The minimax solution minimizes the maximum expected point-loss regardless of an opponent's strategy in zero-sum games. In the game of H_∞ filtering, the filter designer prepares for the worst case that the opponent player can provide. In other words, the goal of the filter is to obtain constantly stable and small estimation errors avoiding divergence from a true value over the state space (K) against any combinations of state process noise, measurement noise, and any initial state. Therefore, the maximizer tries to give the combination of "the worst disturbance" and "the worst initial error condition" while the minimizer obtains the optimal estimates. Consequently, H_∞ filtering does not require the prior knowledge of noise statistics, and deals with deterministic noisy disturbance in its algorithm (Shen & Deng, 1997) unlike the method based on the KF. While the KF minimizes MSE of the estimate, H_∞ filtering is designed to minimize the possible worst error (Simon, 2006)(Lim, 2014).

Therefore, "MSE" and the "possible worst error" are defined as the risk functions to be minimized in the KF and H_∞ filtering, respectively.

In H_∞ filtering, the estimator, i.e. numerator of Eq. (12) below, plays against the exogenous noises and the initial state uncertainty, i.e. the denominator of Eq. (12). Accordingly, the risk function for the designer in H_∞ filtering is defined as follows:

$$J = \frac{\sum_{k=0}^{K} \| \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k \|^2_{\boldsymbol{\chi}_k}}{\| \boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0 \|^2_{\check{\boldsymbol{P}}_0^{-1}} + \sum_{k=0}^{K} \left(\| \boldsymbol{u}_k \|^2_{\boldsymbol{W}_k^{-1}} + \| w_k \|^2_{\boldsymbol{V}_k^{-1}} \right)}, \tag{12}$$

where K is the number of total time steps, $\hat{\boldsymbol{x}}_k$ is the estimated state at time step k, $\boldsymbol{u}_k$ is the state noise, w_k is the observation noise, and $\boldsymbol{x}_0$ is the initial state, respectively. $\boldsymbol{\chi}_k$, $\check{\boldsymbol{P}}_0$, $\boldsymbol{W}_k$ and $\boldsymbol{V}_k$ are $\eta \times \eta$ weighting matrices for all estimates errors, initial estimate error, process noise, measurement noise, respectively, and $\| \cdot \|$ denotes the vector norm, i.e. $\| \boldsymbol{u}_k \|^2_{\boldsymbol{W}_k^{-1}}$ implies $\boldsymbol{u}_k^\top \boldsymbol{W}^{-1} \boldsymbol{u}_k$ where $\top$ denotes the matrix transpose. The way how the weighting matrices are determined is that, for instance, if it is known that the second element of $\boldsymbol{u}_k$ is small, then $(2, 2)$ entry of $\boldsymbol{W}_k$ is chosen to be small compared to other elements. In our work, we assume that $\boldsymbol{V}_k$ and $\boldsymbol{W}_k$ are time invariant while they can be time variant.

Maximal J is minimized with a bound γ as follows:

$$\sup J < \gamma^{-1}, \tag{13}$$

where "sup" denotes the "supremum," and $\gamma > 0$ is a predetermined level of noise (disturbance) attenuation. Therefore, a large γ means a small level of noise, and our maximization of J is limited to the reciprocal of γ. Then, $\bar{J}$ is defined as

$$\bar{J} = -\gamma^{-1} \parallel \boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0 \parallel^2_{\check{\boldsymbol{P}}_0^{-1}} + \sum_{k=0}^{K} \left[\parallel \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k \parallel^2_{\boldsymbol{\chi}_k} - \gamma^{-1} \left(\parallel \boldsymbol{u}_k \parallel^2_{\boldsymbol{W}_k^{-1}} + \parallel w_k \parallel^2_{\boldsymbol{V}_k^{-1}} \right) \right],$$

and the problem becomes the following minimax problem:

$$\min_{\hat{\boldsymbol{x}}_k} \left(\max_{\boldsymbol{u}_k, w_k, \boldsymbol{x}_0} \bar{J} \right). \tag{14}$$

The H_∞ filter recursively solves the above minimax problem, sequentially computing the coefficients $\hat{\boldsymbol{x}}_k$ via

$$\hat{\boldsymbol{x}}_k = \boldsymbol{F}\hat{\boldsymbol{x}}_{k-1} + \boldsymbol{h}_k \left(y_k - \boldsymbol{g}_k \hat{\boldsymbol{x}}_{k-1} \right) \tag{15}$$

where the $\eta \times 1$ vector $\boldsymbol{h}_k$ is the $\boldsymbol{H}_\infty$ gain, to be defined below (Shen & Deng, 1997). Such a solution exists if and only if there exists a symmetric $\eta \times \eta$ matrix $\check{\boldsymbol{P}}_{k+1} > 0$ for the following Riccati equation:

$$\check{\boldsymbol{P}}_{k+1} = \boldsymbol{F}\check{\boldsymbol{P}}_k \boldsymbol{S}_k \boldsymbol{F}^\top + \boldsymbol{W}_k, \tag{16}$$

where $\boldsymbol{F}$ is an $\eta \times \eta$ identity matrix in our problem. The H_∞ gain is defined as

$$\boldsymbol{h}_k = \boldsymbol{F}\check{\boldsymbol{P}}_k \boldsymbol{S}_k \boldsymbol{g}_k{}^\top \boldsymbol{V}_k^{-1}, \tag{17}$$

where $\eta \times \eta$ matrix

$$\boldsymbol{S}_k = \left(\boldsymbol{I} - \gamma \boldsymbol{\chi}_k \check{\boldsymbol{P}}_k + \boldsymbol{g}_k{}^\top \boldsymbol{V}_k^{-1} \boldsymbol{g}_k \check{\boldsymbol{P}}_k\right)^{-1}, \tag{18}$$

such that $\boldsymbol{I}$ is the $\eta \times \eta$ identity matrix. If we adopt an identity matrix for $\boldsymbol{\chi}_k$, the maximum value of the bound γ is constrained by the condition as follows. From Eqs. (16) and (18), γ should satisfy

$$\check{\boldsymbol{P}}_k \left(\boldsymbol{I} - \gamma \boldsymbol{I} \check{\boldsymbol{P}}_k + \boldsymbol{g}_k{}^\top \boldsymbol{V}_k^{-1} \boldsymbol{g}_k \check{\boldsymbol{P}}_k\right)^{-1} > -\boldsymbol{W}_k. \tag{19}$$

Then, from Eq. (13), we can obtain $0 < \gamma < \gamma_{\max}$, where

$$\gamma_{\max} \boldsymbol{I} = \check{\boldsymbol{P}}_k^{-1} + \boldsymbol{g}_k{}^\top \boldsymbol{V}_k^{-1} \boldsymbol{g}_k + \boldsymbol{W}_k^{-1} \tag{20}$$

to maintain $\check{\boldsymbol{P}}_{k+1} > 0$ in the Riccati equation of Eq. (16) (Shen & Deng, 1997). Because the right hand side of Eq. (20) is time varying, γ needs to be carefully selected, and usually a very small value is employed. We select γ based on preliminary training process for optimal performance of the filter. The training process is required to find the values of various parameters in accordance mainly with the noise statistics that enables the filter to perform optimally. The specific steps of H_∞ filtering algorithm for estimating $\boldsymbol{x}$ are summarized in the following.

- **Initialization** Initialize the performance bound γ, the estimate $\hat{\boldsymbol{x}}_0$, $\check{\boldsymbol{P}}_0$, and weight parameters ($\boldsymbol{\chi}_k$, $\boldsymbol{W}_k$, $\boldsymbol{V}_k$).

- **Recursive update** for $k = 1, \ldots, K$

  1. Compute H_∞ gain:

     $\boldsymbol{h}_k = \boldsymbol{F}\check{\boldsymbol{P}}_k\boldsymbol{S}_k\boldsymbol{g}_k{}^\top\boldsymbol{V}_k^{-1}$ where $\boldsymbol{S}_k$ is obtained from Eq. (18).

  2. Update the estimate:

     $\hat{\boldsymbol{x}}_k = \boldsymbol{F}\hat{\boldsymbol{x}}_{k-1} + \boldsymbol{h}_k\,(y_k - \boldsymbol{g}_k\hat{\boldsymbol{x}}_{k-1})$.

  3. Update the error covariance:

     $\check{\boldsymbol{P}}_{k+1} = \boldsymbol{F}\check{\boldsymbol{P}}_k\boldsymbol{S}_k\boldsymbol{F}^\top + \boldsymbol{W}_k$.

4. Performance Assessment via Experiments

We assess the performance of the proposed approach by using real meteorological data provided by Korean Meteorological Administration (KMA). We perform experiments with irradiance data while approaches are applicable for any other meteorological data as long as NWP is employed. Local Data Assimilation and Prediction System (LDAPS) of the Unified Model (UM) NWP model was used for our investigation. The weather variable for the irradiance we used is downward short wave radiation flux (DSWRF) at the surface. Irradiance values are divided by $1,000$ W/m^2 for filtering experiments.

4.1 Preliminary Experiment

As preliminary experiments, we applied the approach to data measured at the Chuncheon Meteorological Observatory for a period of one month. We consider two specific forecast horizons. That is, for an 1 hour forecast horizon we study irradiance at 10 a.m., and for a 24 hour forecast horizon we study irradiance at 3 p.m. We select these horizons because 24 hours ahead forecast is a typical prediction and 1 hour is just for comparison purpose. We consider degrees 1 and 2 polynomial functions. The data of April and May in 2014 are used for bias correction of the dates from May 2 to 31 in 2014. Therefore, $K = 30$ is used.

Specifically, if we want to estimate the bias of 1h forecast of NWP for irradiance at 10 a.m., May 1 with $K = 30$, we need forecasted data of NWP at 9 a.m. from April 1 to April 30. Besides, we also need the measured data at 10 a.m. from April 1 to April 30. Then, we subtract the measurement data from the NWP outputs to obtain y_k, where $k = 1, \ldots, 30$. With given initial $\hat{\boldsymbol{x}}_0$ (whose all elements are zeros), we sequentially estimate $\hat{\boldsymbol{x}}_k$, where $k = 1, \ldots, 30$. Finally, we use $\hat{\boldsymbol{x}}_{30}$ to make $\boldsymbol{g}_{30} \cdot \hat{\boldsymbol{x}}_{30}$ in order to obtain corrected forecast of NWP for 10 a.m. on May 1 by $m_{\mathrm{May1},f} - \boldsymbol{g}_{30} \cdot \hat{\boldsymbol{x}}_{30}$ as described in the block diagram of Fig. 1 where $m_{\mathrm{May1},f}$ denotes the 1h irradiance forecast via NWP for 10 a.m. on May 1. In the case of 24h prediction, the description is similar, but slightly different in terms of employed data. If we want to estimate the bias of 24h forecast of NWP for irradiance at 3 p.m. on May 2 with $K = 30$, we need forecasted data of NWP at 3 p.m. from April 1 to April 30. Besides, we also need the measured data at 3 p.m. from April 2 to May 1. Then, we subtract the measured data from the NWP data to obtain the biases used as y_k in the measurement equation Eq. (6), where $k = 1, \ldots, 30$. With given initial $\hat{\boldsymbol{x}}_0$, we sequentially estimate $\hat{\boldsymbol{x}}_k$, where $k = 1, \ldots, 30$. Finally, we use $\hat{\boldsymbol{x}}_{30}$ to make $\boldsymbol{g}_{30} \cdot \hat{\boldsymbol{x}}_{30}$ in order to obtain corrected forecast of NWP for 3 p.m. on May 2 by $m_{\mathrm{May2},f} - \boldsymbol{g}_{30} \cdot \hat{\boldsymbol{x}}_{30}$ as described in the block diagram of Fig. 1 where $m_{\mathrm{May2},f}$ denotes the 24h irradiance forecast of NWP for 3 p.m. on May 2 in this case.

Let us define required notations first as follows: D is the number of predicted days; D_P is the date range associated with D for the computation of mean absolute error, the maximum error, and for tuning parameters in H_∞ filtering; $D_{k,f}$, for $k = 1, \ldots, D$ is the date range of forecasted days in NWP to predict the bias of the k-th day of D_P; therefore, $D_{1,f}$ is the date range of forecasted days in NWP for the prediction of the first day of D_P; $D_{k,r}$, for $k = 1, \ldots, D$ is the date range of measured days for the prediction of the k-th day of D_P; therefore, $D_{1,r}$ is the date range of measured days for the prediction of the first day of D_P; respectively. The rest of $D_{k,f}$ and $D_{k,r}$, can be obtained by shifting the date range by day differences from

$D_{1,f}$ and $D_{1,r}$, respectively.

Therefore, given D and D_P, we have D pairs of different date ranges (both forecasted and measured data) for training $\hat{\boldsymbol{x}}_k$. Nevertheless, we tune the parameters of H_∞ filter only once based on mean absolute error, and the maximum error with respect to whole D days and associated D_P. Therefore, the parameters of H_∞ filter are tuned over the date range D_P. We initialize parameters for H_∞ filtering based on training process as shown in Table 1, where $\check{\boldsymbol{P}}_0 = \rho\boldsymbol{I}$, $\boldsymbol{W}_0 = \omega\boldsymbol{I}$, respectively. We begin with $\boldsymbol{x}_0 = [0\ 0]^\top$ in the case of the KF for the degree 1 polynomial function, and $\boldsymbol{x}_0 = [0\ 0\ 0]^\top$ for degree 2, respectively. Table 1

For the KF, as suggested in (Pelland, Galanis & Kallos, 2013), we initialize parameters, and compute noise, variance, and covariance as follows: $\boldsymbol{x}_0 = [0\ \ 0]^\top$for $n = 1; y_0 = 0; \boldsymbol{P}_0 = 5 \times 10^{-5}\boldsymbol{I}; \boldsymbol{Q}_{u,0} = 10^{-5}\boldsymbol{I}; q_{w,0} = 0.01; \boldsymbol{u}_k = \hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_{k-1}; w_k = y_k - \boldsymbol{g}_k\hat{\boldsymbol{x}}_k; \boldsymbol{Q}_{u,k+1} =$ sample covariance of $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_k\}$; $q_{w,k+1} =$ sample variance of $\{w_1, w_2, \ldots, w_k\}$, where the initial values were obtained via the training of real data for an year by the authors of (Pelland, Galanis & Kallos, 2013).

Table 2 shows the results for Chuncheon for a period from May 1 to 31 in 2014 by the proposed H_∞ filtering and the KF in various scenarios along with NWP predictions, where the results show mean absolute error, the maximum absolute error, and bias (i.e. mean bias error). From the results, the proposed H_∞ filtering shows significantly improved bias prediction accuracy beyond the bias-mitigation by the KF. Generally, all methods show better performance for 1h prediction than that for 24h prediction. Both filters show better performance when $n = 2$ than that when $n = 1$ in most cases except for the mean absolute error by H_∞ filter. Figs. 2-3 show time series of absolute error when n=1 that correspond to the results of Table 2. It is clear that the maximum error for the H_∞ filter is actually smaller than that of the KF. In Fig. 2, the H infinity filter outperforms the KF when the absolute error is small. However, in Fig. 3, it is not the case. Although we can obtain better performance of H_∞ filter than that of the KF overall, we may not able to obtain outperforming results of H_∞ filter over KF over all range of time. Our goal is focused on minimizing the maximum error via H_∞ filter. Fig. 2

Fig. 3

4.2 Further Investigation Table 2

We further investigate the proposed approach via extensive experiments. Five cities in Korea are investigated as listed in Table 3 where their geographical coordinates are shown. Firstly, we set $n = 1$, and tune the weighting parameters with respect to each horizon and three window sizes, i.e. 30, 60, and 90, respectively. Besides, a longer time period from March to December of 2014 is investigated; therefore, it is predicted for $7 \sim 9$

Table 3

months depending on the window size, that is, 7 months if we take $K = 90$ and 9 months for $K = 30$, respectively. Specific values of D and D_P are shown in Table 4. Therefore, after extensive experiments of tuning process, we customize tuned parameter settings depending on locations as shown in Table 4. We observe that the tuned parameters for Chuncheon is different depending on investigated time period and window size K as shown in Tables 1 and 4. Then, with determined tuned parameters, we investigate the performance of H_∞ filter as the function of the degree of polynomial function as shown in Figs. 4-5 where root mean square error (RMSE) improvement over NWP is shown and that of the KF was also compared in the case of Jeonju. Figs. 4-5 show that, generally, the RMSE improvement is either increased or decreased (overfitting) in accordance with increasing n while eventually it stabilizes. The rest of the cities show similar results of patterns as in Figs. 4-5 that the improvement in RMSE appears to be either increased or decreased (overfitting) in accordance with the increasing polynomial degree before it finally stabilizes, regardless of the filter type and the size of K. While the variation of RMSE improvement manifests clearly in accordance with increasing n, the absolute difference is not significant, and both filters undergo increased computational complexity if n increases with the risk of overfitting.

Table 4

Fig. 4

Fig. 5

Finally, we showed the result with customized tuned parameters, K, and n depending on locations in Table 5. For instance, for 24 hours horizon at Seosan observatory, $K = 30$, $n = 1$ for H_∞ filtering while $K = 60$, $n = 1$ were selected for the Kalman filtering. In all cases, the proposed H_∞ filtering outperforms the KF in terms of mean absolute error, the maximum absolute error, and bias. All biases are less than 0.6% and greater than -0.6 %. As shown in the results, generally, we can obtain smaller error for 1h horizon than that for 24h horizon.

Table 5

We also show the superiority of H_∞ filtering to the KF in terms of computational complexity as shown in Fig. 6. Fig. 6 shows mean values (s) of MATLAB processing time of from seven to nine months over $1,000$ runs of experiments. It takes a longer time for the larger K because it requires more number of training days. Elapsed time is also increased in accordance with increasing n. As shown in Fig. 6, H_∞ filtering requires significantly less processing time compared to that of the Kalman filtering. Although we showed the result only for 1 hour horizon prediction of Chuncheon, the rest of locations showed similar results including the 24 hour prediction cases.

Fig. 6

The KF assumes that noise statistics of the state and the measurement equations are given regardless of their correctness. Therefore, if we want to apply the KF to practical problems, we have to guess or estimate the noise statistics because the statistics are not usually known. In this problem, sample mean and variance are calculated for the KF while weighting parameters and γ are trained for H_∞ filtering. Particularly, training γ is crucial for satisfactory performance of the filter. Besides, the proposed H_∞ filtering is simpler than the KF in terms of computational complexity of the algorithm as shown in Fig. 6. Therefore, the proposed H_∞ filtering has significant advantages over the KF to solve practical problems where exact information of noise statistics is unknown.

5. Conclusion

In this paper, we proposed a new approach to the problem of predicting biases of NWP based on post-processing of past data of both predictions and real measurements. The proposed approach does not require the information of noise statistics while the Kalman algorithm requires it. While the KF is the optimal approach for linear and Gaussian problems, we may not obtain optimal solution by employing the KF because many things are unknown to solve practical problems such as bias correction. Therefore, we obtained outperforming result of the proposed H_∞ filtering over the KF in this problem, particularly by reducing large errors. In terms of computational complexity, it requires less number of steps compared to that of the KF. Therefore, the proposed H_∞ filtering is more pertinent for this practical weather prediction problem in terms of estimation accuracy and computational complexity.

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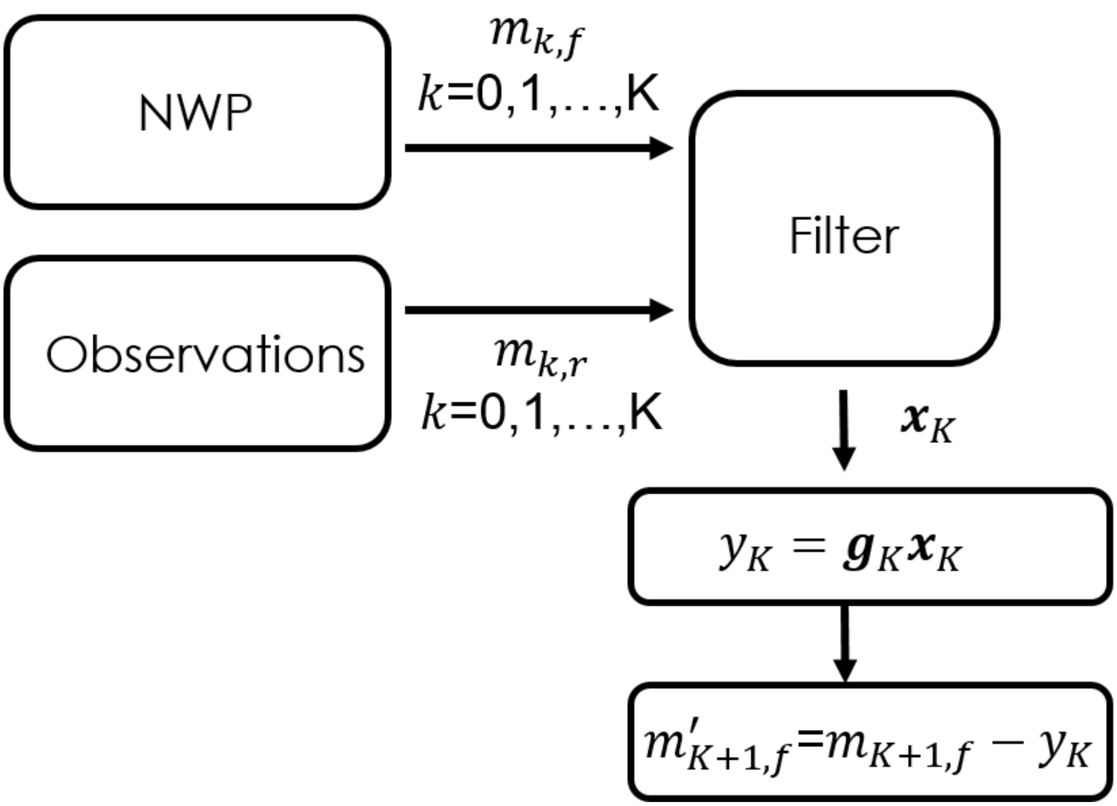

Fig. 1. Block diagram of post-processing by a filter such as the KF or H_∞ filter. An NWP output of $m_{K+1,f}$ is corrected by y_K, and updated as $m'_{K+1,f}$.

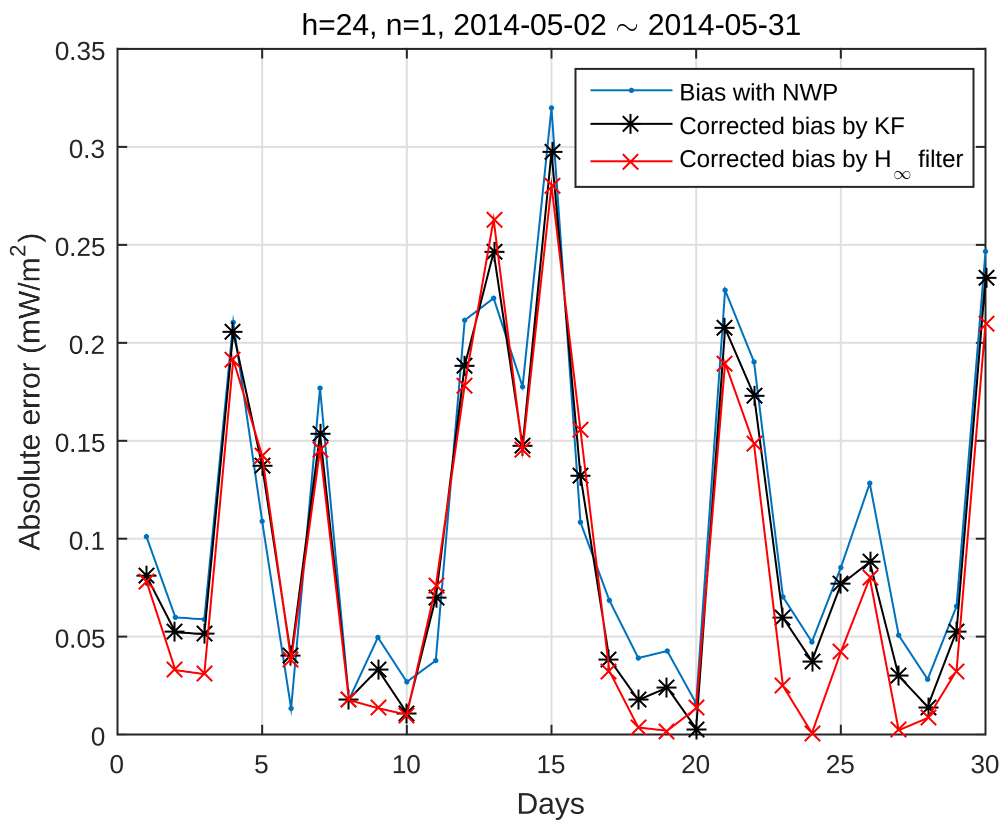

Fig. 2. Absolute errors for predicted irradiance at the Chuncheon Meteorological Observatory. 1-hour forecast horizon at 10 a.m. from May 1 to 30, 2014; $n = 1$.

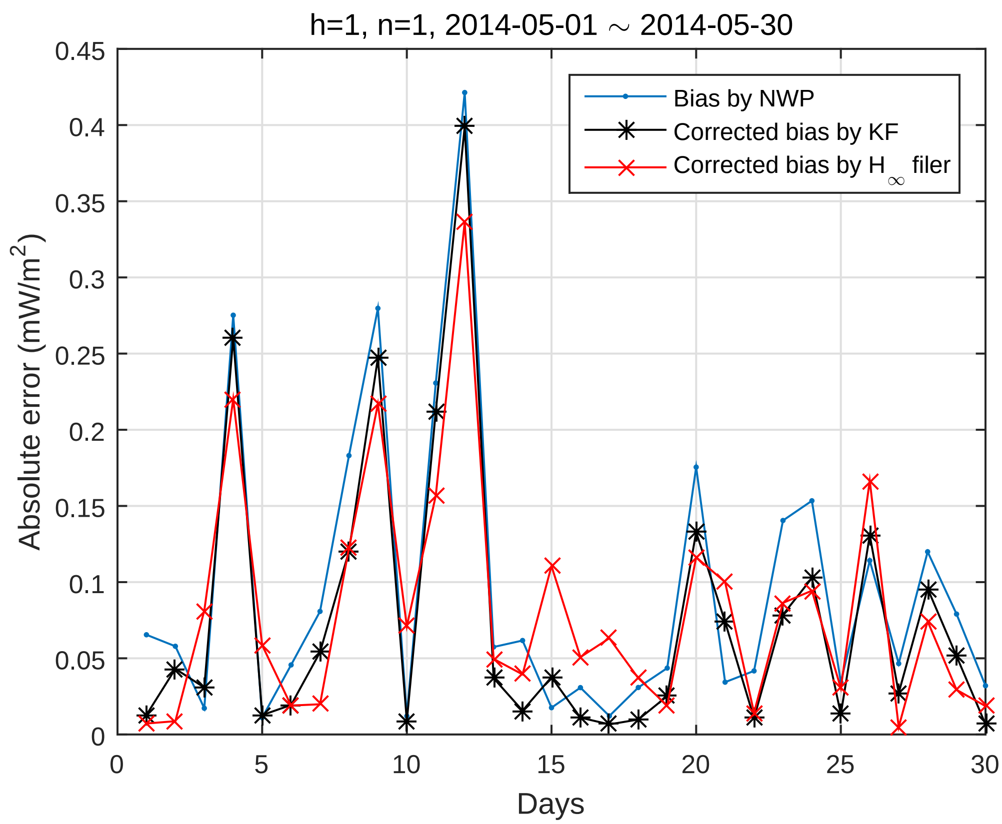

Fig. 3. Absolute errors for predicted irradiance at the Chuncheon Meteorological Observatory. 1-hour forecast horizon at 10 a.m. from May 1 to 30, 2014; $n = 1$.

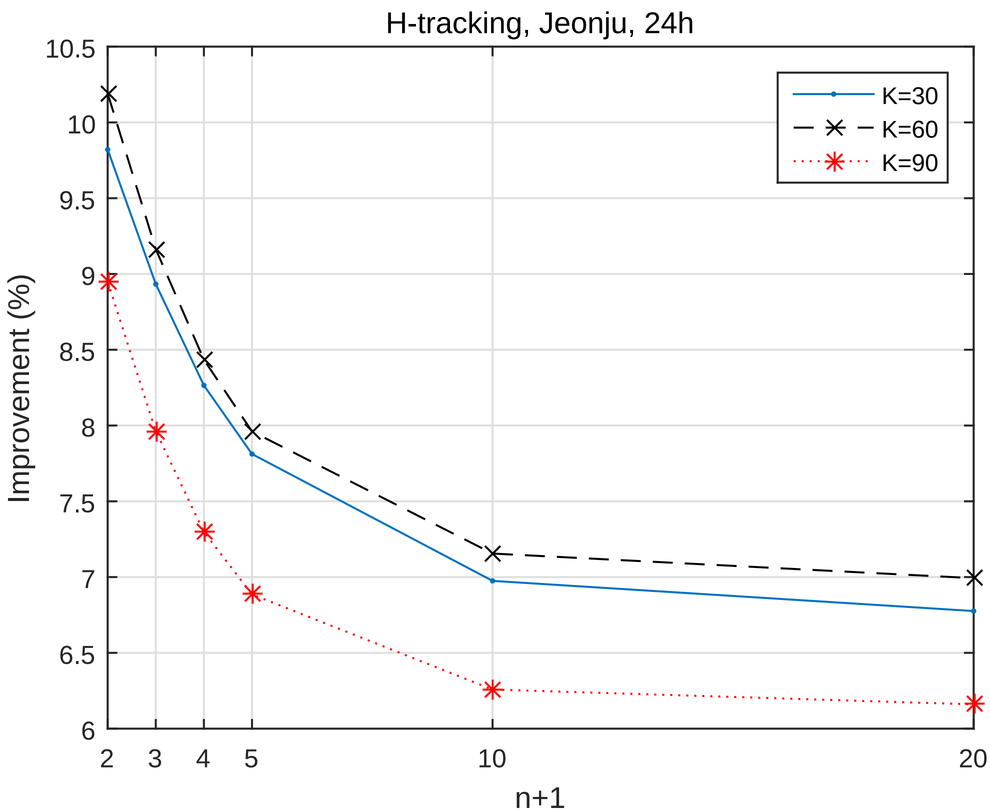

Fig. 4. RMSE reduction in accordance with increasing n compared to NWP at Jeonju for 24-hour forecast horizon was depicted. Results for the KF was shown for three window sizes, i.e., 30, 60, and 90.

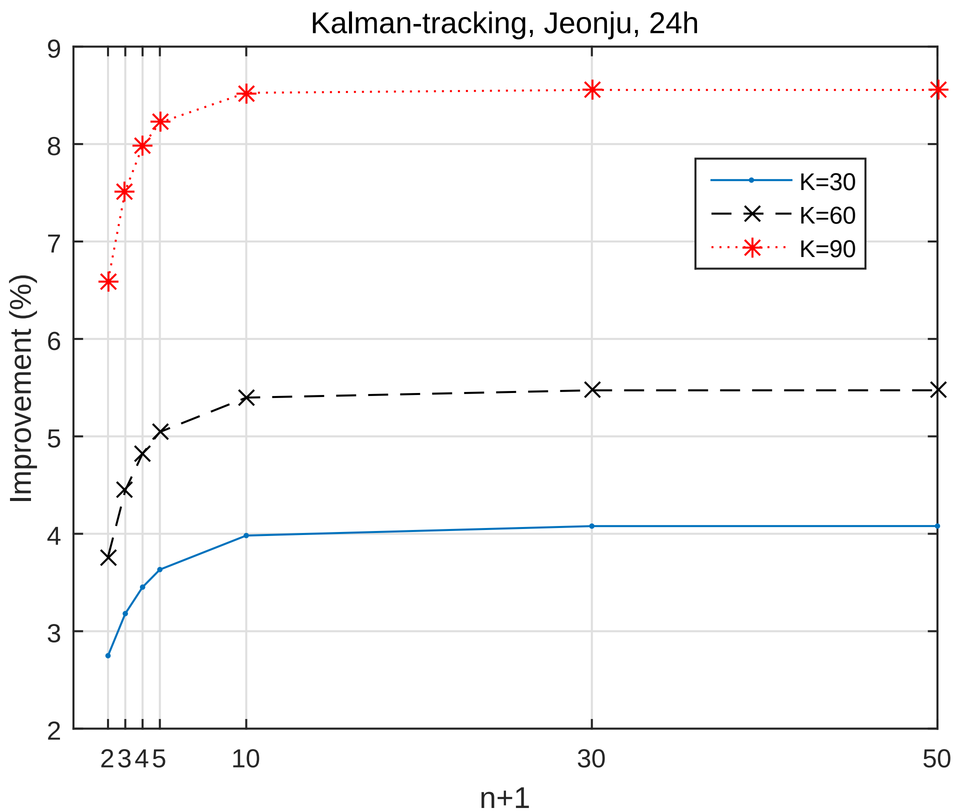

Fig. 5. RMSE reduction in accordance with increasing n compared to NWP at Jeonju for 24-hour forecast horizon was depicted. Result for the H_∞ filter was shown for three window sizes, i.e., 30, 60, and 90.

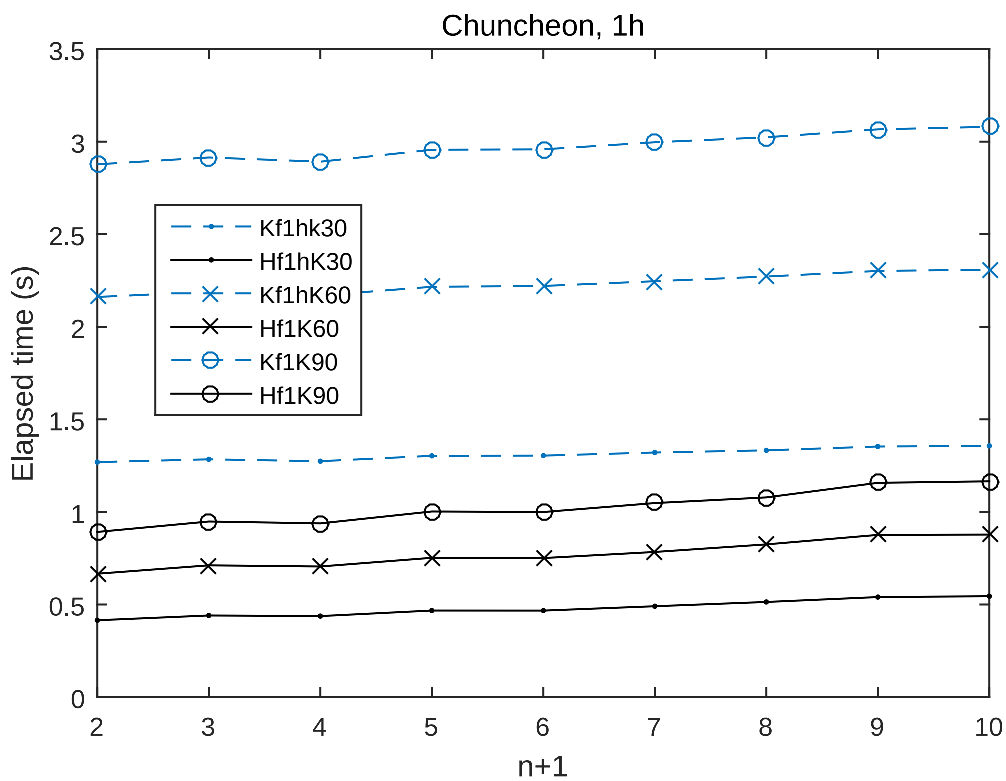

Fig. 6. The elapsed processing time of the methods based on H_∞ and Kalman filters for 1-hour prediction at Chuncheon, where Kf1hK30 denotes the Kalman filtering, 1-hour prediction, $K = 30$, and Hf1hK30 denotes H_∞ filtering, 1-hour prediction, $K = 30$. The rest of legends are similarly described. The mean values of MATLAB processing time from seven to nine months over 1000 runs of experiments.

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Table 1. Tuned parameters of H_∞ filtering for the Chuncheon Meteorological Observatory. Associated D (the number of predicted days) and D_P (date range for D) are also specified.

Location	Hrzn	K	n	γ	V_0	ρ	ω	D	D_P
Chuncheon	24h	30	1	0.1	0.2	5×10^{-3}	1×10^{-4}	30	2014-05-02 ∼ 2014-05-31
		30	2	0.1	0.2	5×10^{-3}	1×10^{-5}		
	1h	30	1	2	0.5	1×10^{-3}	5×10^{-4}	30	2014-05-01 ∼ 2014-05-30
		30	2	2	0.5	1×10^{-3}	5×10^{-4}		

Table 2. Summary of the results for the Chuncheon Meteorological Observatory, where K, n, Kf, Hf, N denote the window size, polynomial order, Kalman filter, H_∞ filter, NWP, respectively. The reduction of mean absolute error was computed by the reduction ratio compared to that of NWP. The maximum absolute biases over 1 month are also shown, where the unit of all values is mW/m^2. Averaged irradiance values for 24 and 1-hour forecast horizons are shown at the third column. Bias means mean bias error of filters relative to the Avg. value.

Location	Hrzn	Avg.	K	n	Mean absolute error		Reduction	Maximum absolute error		Bias
Chuncheon	24h	0.5603	30	1	N: 0.1143	Kf: 0.0966	15.49 %	N: 0.4606	Kf: 0.6218	0.07 %
			30	2	N: 0.1143	Kf: 0.0966	15.48 %	N: 0.4606	Kf: 0.4340	0.09 %
			30	1	N: 0.1143	Hf: 0.0947	17.14 %	N: 0.4606	Hf: 0.4104	0.03 %
			30	2	N: 0.1143	Hf: 0.0948	17.04 %	N: 0.4606	Hf: 0.4054	-0.07 %
	1h	0.4280	30	1	N: 0.0968	Kf: 0.0763	21.26 %	N: 0.4214	Kf: 0.3989	-0.20 %
			30	2	N: 0.0968	Kf: 0.0753	22.26 %	N: 0.4214	Kf: 0.3973	-0.16 %
			30	1	N: 0.0968	Hf: 0.0760	21.49 %	N: 0.4214	Hf: 0.3683	0.17 %
			30	2	N: 0.0968	Hf: 0.0762	21.37 %	N: 0.4214	Hf: 0.3656	-0.18 %

Table 3. Geographic coordinates of five locations in Korea for extensive experiments.

Location	Longitude (degree)	Latitude (degree)	Height (m)
Seosan	126.4939	36.7766	28.91
Jeonju	127.1190	35.8408	61.40
Jinju	128.0400	35.1638	30.21
Chuncheon	127.7357	37.9026	76.47
Andong	128.7037	36.5729	140.10

Table 4. Tuned parameters for H_∞ filtering depending on locations, horizons (Hrzn), and K with $n = 1$. Associated D (the number of predicted days) and D_P (date range for D) are also specified. D and D_P compose a pair that provides the same values as long as both the horizon and the window size K are the same.

Location	Hrzn	K	n	γ	V_0	ρ	ω	D	D_P
Seosan	24h	30	1	0.75	0.09	10^{-3}	10^{-4}	275	2014-04-01 ∼ 2014-12-31
		60	1	1	0.1	0.005	5×10^{-4}	245	2014-05-01 ∼ 2014-12-31
		90	1	10^{-3}	0.2	0.03	10^{-3}	215	2014-05-31 ∼ 2014-12-31
	1h	30	1	3	2	10^{-4}	10^{-3}	276	2014-03-31 ∼ 2014-12-31
		60	1	0.7	0.1	10^{-6}	7×10^{-4}	246	2014-04-30 ∼ 2014-12-31
		90	1	10^{-4}	0.1	10^{-6}	6×10^{-4}	216	2014-05-30 ∼ 2014-12-31
Jeonju	24h	30	1	0.1	2	0.035	2.3×10^{-4}	275	2014-04-01 ∼ 2014-12-31
		60	1	10^{-4}	2.12	0.036	3.7×10^{-3}	245	2014-05-01 ∼ 2014-12-31
		90	1	10^{-5}	2.81	0.02	3.8×10^{-3}	215	2014-05-31 ∼ 2014-12-31
	1h	30	1	10^{-4}	1.4	1	10^{-7}	276	2014-03-31 ∼ 2014-12-31
		60	1	0.8	0.51	0.01	3×10^{-5}	246	2014-04-30 ∼ 2014-12-31
		90	1	0.5	3	0.014	2×10^{-4}	216	2014-05-30 ∼ 2014-12-31
Jinju	24h	30	1	1	2.1	0.05	5×10^{-5}	275	2014-04-01 ∼ 2014-12-31
		60	1	10^{-4}	1.7	0.08	10^{-7}	245	2014-05-01 ∼ 2014-12-31
		90	1	10^{-4}	2.1	0.03	2.5×10^{-4}	215	2014-05-31 ∼ 2014-12-31
	1h	30	1	0.1	10^{4}	0.01	10^{-3}	276	2014-03-31 ∼ 2014-12-31
		60	1	1	10^{2}	0.01	10^{-3}	246	2014-04-30 ∼ 2014-12-31
		90	1	10^{-2}	2	0.1	10^{-7}	216	2014-05-30 ∼ 2014-12-31
Chuncheon	24h	30	1	2.4	1.8	0.004	4×10^{-4}	275	2014-04-01 ∼ 2014-12-31
		60	1	10^{-4}	1.5	0.003	5×10^{-4}	245	2014-05-01 ∼ 2014-12-31
		90	1	10^{-4}	10^{5}	70	10^{-4}	215	2014-05-31 ∼ 2014-12-31
	1h	30	1	10^{-3}	2	10^{-3}	5×10^{-4}	276	2014-03-31 ∼ 2014-12-31
		60	1	10^{-5}	0.44	3×10^{-4}	5×10^{-4}	246	2014-04-30 ∼ 2014-12-31
		90	1	10^{-4}	1.1	10^{-5}	5×10^{-4}	216	2014-05-30 ∼ 2014-12-31
Andong	24h	30	1	0.71	0.75	0.044	1.65×10^{-4}	275	2014-04-01 ∼ 2014-12-31
		60	1	1.33	0.1	10^{-7}	4×10^{-4}	245	2014-05-01 ∼ 2014-12-31
		90	1	10^{-5}	0.173	10^{-7}	4.75×10^{-4}	215	2014-05-31 ∼ 2014-12-31
	1h	30	1	10^{-3}	2	1	10^{-4}	276	2014-03-31 ∼ 2014-12-31
		60	1	1	1	0.01	10^{-4}	246	2014-04-30 ∼ 2014-12-31
		90	1	0.9	0.9	0.01	10^{-4}	216	2014-05-30 ∼ 2014-12-31

Table 5. Results of prediction based on the data (both forecasted and measured) of nine months, where Hrzn, K, n, Kf, Hf, N denote the horizon, window size, polynomial order, Kalman filter, H_∞ filter, NWP, respectively. The reduction of mean absolute error was computed by the reduction ratio compared to that of NWP. The maximum absolute biases over associated D_P are also shown, where the unit of all values is mW/m^2. Averaged irradiance values for 24 and 1-hour forecast horizons are shown at the third column. Bias means mean bias error of filters relative to the Avg. value.

Location	Hrzn	Avg.	K	n	Mean absolute error		Reduction	Maximum absolute error		Bias
Seosan	24h	0.4133	60	1	N: 0.1512	Kf: 0.1486	1.72 %	N: 0.6202	Kf: 0.6218	0.56 %
			30	1	N: 0.1487	Hf: 0.1451	2.48 %	N: 0.6202	Hf: 0.6085	-0.54 %
	1h	0.3147	60	1	N: 0.1181	Kf: 0.1119	5.25 %	N: 0.5115	Kf: 0.5273	0.55 %
			30	1	N: 0.1199	Hf: 0.1098	8.35 %	N: 0.5115	Hf: 0.5077	0.52 %
Jeonju	24h	0.4328	90	1	N: 0.2001	Kf: 0.1856	7.22 %	N: 0.8490	Kf: 0.7897	-0.69 %
			30	1	N: 0.2009	Hf: 0.1791	10.86 %	N: 0.8490	Hf: 0.7545	0.68 %
	1h	0.3450	60	1	N: 0.1388	Kf: 0.1364	1.69 %	N: 0.5224	Kf: 0.5682	-0.67 %
			30	1	N: 0.1388	Hf: 0.1351	2.67 %	N: 0.5224	Hf: 0.5222	0.65 %
Jinju	24h	0.4705	30	1	N: 0.1124	Kf: 0.1080	3.91 %	N: 0.6837	Kf: 0.6893	0.31 %
			60	1	N: 0.1113	Hf: 0.1032	7.28 %	N: 0.6837	Hf: 0.6771	-0.38 %
	1h	0.3756	37	1	N: 0.0915	Kf: 0.0910	0.55 %	N: 0.6401	Kf: 0.6377	-0.42 %
			90	1	N: 0.0915	Hf: 0.0899	1.67 %	N: 0.6401	Hf: 0.6374	0.41 %
Chuncheon	24h	0.4435	30	1	N: 0.1049	Kf: 0.1032	1.62 %	N: 0.5816	Kf: 0.5935	0.22 %
			30	1	N: 0.1049	Hf: 0.1023	2.48 %	N: 0.5816	Hf: 0.5803	-0.33 %
	1h	0.3098	30	1	N: 0.0898	Kf: 0.0852	5.12 %	N: 0.4560	Kf: 0.5020	-0.43 %
			30	1	N: 0.0898	Hf: 0.0845	5.90 %	N: 0.4560	Hf: 0.4424	0.42 %
Andong	24h	0.5071	30	1	N: 0.1145	Kf: 0.1104	3.58 %	N: 0.6763	Kf: 0.6698	-0.20 %
			30	1	N: 0.1145	Hf: 0.1079	5.76 %	N: 0.6763	Hf: 0.5989	-0.20 %
	1h	0.3901	30	1	N: 0.1150	Kf: 0.1123	2.35 %	N: 0.4717	Kf: 0.4746	0.48 %
			30	1	N: 0.1150	Hf: 0.1092	5.00 %	N: 0.4717	Hf: 0.4703	-0.44 %